

DOCUMENT RESUME

ED 045 448

SP 010 448

TITLE The Teaching of Modern Mathematics, K-6
[Instructional Model, Curriculum Guide, Scope and
Sequence Guide and Teacher's Manual].

INSTITUTION California Area School District, Pa.

SPONS AGENCY Bureau of Elementary and Secondary Education
(OHEW/OE), Washington, D.C.

DATE 60

NOTE 157p.

EDRS PRICE MF-\$0.75 PC Not Available from EDRS.

DESCRIPTORS Curriculum Development, *Curriculum Guides,
*Elementary School Mathematics, Instruction,
*Instructional Materials, Modern Mathematics

IDENTIFIERS ESFA Title III

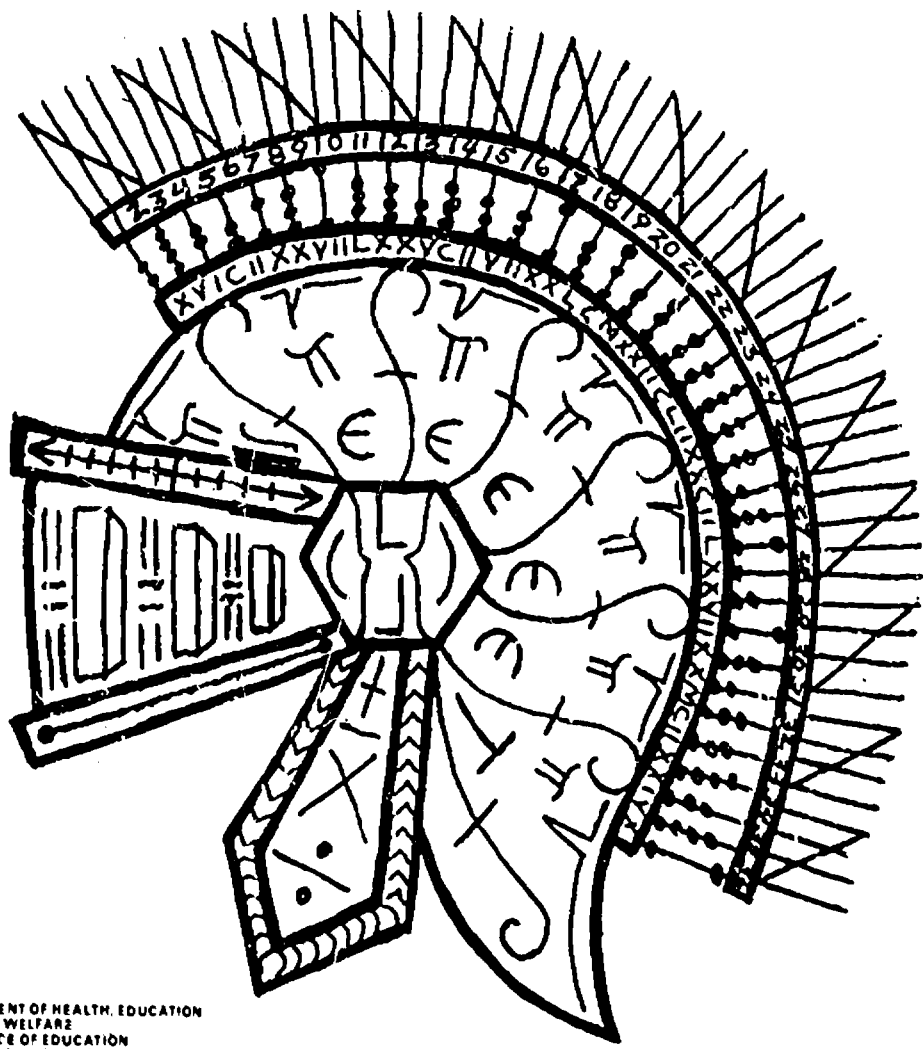
ABSTRACT

Included in this set of materials is a description of an instructional model for elementary school mathematics, a curriculum guide developed by the elementary school teachers of the California Area School District, scope and sequence guides, and a teachers manual. The instructional model includes the use of pretests, posttests, prerequisites and behaviors as they relate to the idea of flexible grouping in elementary school mathematics. The teachers manual contains materials for each content item described in the curriculum guide as well as lesson plans, worksheets, prerequisites and content behaviors, pupil response sheets, enrichment materials, and individual record sheets. This work was prepared under an ESFA Title III contract. [Not available in hardcopy due to marginal legibility of original document.] (5L)

ELEMENTARY MATHEMATICS

CALIFORNIA AREA SCHOOLS

ED0 45448



U.S. DEPARTMENT OF HEALTH, EDUCATION
& WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED
EXACTLY AS RECEIVED FROM THE PERSON OR
ORGANIZATION ORIGINATING IT. POINTS OF
VIEW OR OPINIONS STATED DO NOT NECESSARILY
REPRESENT OFFICIAL OFFICE OF EDUCATION
POSITION OR POLICY

CURRICULUM GUIDE

SCOPE & SEQUENCE GUIDE

1968 - 1969

ED0 45448

ED0 45448



CURRICULUM DEVELOPMENT

ELEMENTARY MATHEMATICS

CALIFORNIA AREA SCHOOLS

John J. Cairns
Superintendent

Donald Sapko
Mathematics Consultant

Joseph M. Bodnar
Director of
Elementary Education

Dennise Shepley
Project Chairman

This Pilot Project has been made possible through a Title III, ESEA grant made available through operations of The Joint Board of County School Directors.

Washington, Fayette, and Greene
Counties

CURRICULUM DEVELOPMENT

ELEMENTARY MATHEMATICS

CORE COMMITTEE

GRADE

GRADE LEVEL COMMITTEE

Juanita Frederick

K

Janet Albenze

1

Louise Jameson

2

Lena Vella

3

Donald Hormell

4

Thomas Virag

5

Harvey Cornell

6

Helen Buk

Martha Dunegan

Betty Lazzari

Janet Simpson

Jeffrey Weaver

Dolores Kaukonen

**Grade level
committee also
included all
members of the
Core Committee.**

**Donald Sapko
Mathematics Consultant**

**Dennise Shepley
Project Chairman**

The committees wish to thank the Department of Public Instruction, Harrisburg, Pennsylvania for the use of the procedures in curriculum development and materials selection developed by PRIMES.

We are also grateful for the personal assistance of the department staff, Mr. Kenneth Colkin, Miss Doris Creswell, and Mr. Emanuel Berger.

CONTENTS

Objectives of Basic Content Items
Interpretation

PRIMARY SECTION

Scope and Sequence Chart
Vocabulary
Materials
Items Ranked as to Importance
Methodology

INTERMEDIATE SECTION

Scope and Sequence Chart
Vocabulary
Materials
Items Ranked as to Importance
Methodology

OBJECTIVES OF BASIC CONTENT ITEMS

Whole Numbers

To develop a basic understanding of the structure of the system of whole numbers.

To develop the ability to work with the four fundamental processes of arithmetic.

To utilize the basic properties of associativity, commutativity, distributivity, identity, inverse, and closure.

To develop an appreciation of the use of the arithmetic of whole numbers in every day life experiences such as in problem solving, games, banking and other challenging and interesting situations.

Fractions

To develop an intuitive understanding of the concepts of fractions as they relate to real experiences.

To help students interpret the physical world they see around them.

To develop the meanings and methods of computations with fractions in all four fundamental operations.

To develop an appreciation of the value of building every mathematical process on simple, basic number relationships presented early in the students career.

Integers

To expand the students awareness of numbers other than those with which they are familiar.

To develop intuitive background of numbers less than zero in the physical environment.

To develop an understanding of integers so that they might be used in examining the property of closure for whole numbers.

To provide an opportunity for students to reason mathematically and form conclusions consistent with past experiences.

Deeper realization and appreciation of the uses and applications of integers will depend on the child as he matures in his study and understanding of basic mathematical concepts.

Numeration and Notation

To develop an understanding and deeper insight into forms of numeration and notation, such as the concept of numerals (ie other names for a number), positional notation in base ten and other bases, and symbols used in the historical evolution of man's ideas about number.

To apply our basic notational forms to the four fundamental operations, fractions, and decimals in mathematics.

Sets

To develop concepts of, and relationships among sets.

To use sets and set notation as a basic foundation for fundamental ideas in various areas of mathematics.

To appreciate the use of sets in other content areas in the elementary school.

Geometry

To develop intuitively such concepts as point, line, plane, and space.

To explore the various interrelationships through measurement, construction and environment.

To apply geometrical ideas and patterns to the physical world.

To appreciate the artful design in geometric figures and in nature.

Measurement

To develop an understanding of measurement as a form of comparison with standard units.

To involve the students in the subject of chronology giving them some relatively simple ideas that will be useful to them in their study of history.

To develop an appreciation and an awareness of the need for measurements and standard units of measure in our society.

Number patterns and relationships

To develop an awareness of the variety of patterns in our number system.

To provide interesting and challenging situations for the growth of perceptive insight and creative approaches in the analysis of numerous mathematical arrangements.

To apply many numerical and geometric relationships encountered through classroom experiences to activities in everyday life.

To appreciate the structure and the beauty of the designs of nature.

Other topics

To provide an atmosphere conducive to ingenious and creative response to a variety of activities which extend and expand the mathematics program of the elementary school.

To appreciate the vast scope of mathematics which is involved in man's everyday activities.

INTERPRETATION

Scope and Sequence Chart

Important Items

The item number is necessary for easy access and location of content items in the Primes Content Authority List with a complete description and selected definitions.

Each grade level is divided into three month periods designated by:

B - beginning of the year

M - middle of the year

E - end of the year

In these columns will be found either

I - introduction of content item

M - mastery of content item

Thus the reader knows the grade level and approximate time of the year that a particular content item is introduced, and also the grade level and approximate time of the year when the item should be mastered.

**PRIMARY
SECTION**

Item	Description	Kindergarten						First			Second			Third			Mastery Beyond Primary
		B	M	E	E	B	M	E	E	B	M	E	E	B	M	E	
0010-0100	<u>BASIC CONCEPTS OF WHOLE NUMBER.</u>																
0010	<u>Definition: Set of whole numbers.</u>							I									4
0020	<u>Develop cardinal number zero.</u>								M								
0030	<u>Develop cardinal number 1-10.</u>	I							M								
0035	<u>Cardinal number beyond 10.</u>							I						M			
0040	<u>Develop ordinal number sense.</u>				I									M			
0050	<u>Number with number line.</u>				I									M			
0060	<u>Counting-Cardinal number of set.</u>	I							M								
0070	<u>Ordinal counting-sequence.</u>				I									M			
0080	<u>Skip counting.</u>							I									4
0900	<u>Backward-rota counting.</u>				I										M		
0100	<u>Ordering-equal, greater, less.</u>				I												5
0110-0200	<u>PROPERTIES OF ADDITION.</u>																
0120	<u>Union of disjoint sets-joining.</u>				I									M			
0130	<u>Developed from number line.</u>					I									M		
0150	<u>Commutativity.</u>				I												4
0160	<u>Associativity.</u>							I									4
0170	<u>Zero-identity element.</u>							I									5
0180	<u>Property of one.</u>				I									M			
0190	<u>Elementary fact of addition.</u>													M			



ERIC
Full Text Provided by ERIC

PROPERTIES OF ADDITION OF FRACTIONAL NUMBER.

Item	Description	Learning Objectives						First	Second						Total
		B	M	E	F	M	E		B	M	E	F	M	E	
1000-1310	<u>PROPERTIES OF SUBTRACTION WITH FRACTIONS.</u>														
1250	<u>Developed from plane or solid regions.</u>										I				5
1260	<u>Inverse of adding fractions.</u>										I				5
1270	<u>Property of zero.</u>										I				5
1290	<u>Common fractions equal denominators.</u>										I				5
1320-1440	<u>PROPERTIES OF MULTIPLICATION WITH FRACTIONS.</u>														
2000-2040	<u>BASIC NUMBER CONCEPTS WITH INTEGERS.</u>														
2020	<u>Developed from physical world situations.</u>										I				6
2050-2100	<u>PROPERTIES OF ADDITION WITH INTEGERS.</u>														
3000-3160	<u>NUMERATION AND NOTATION.</u>														
3000	<u>Difference between number and numeral.</u>						I								4
3010-3040	<u>DIFFERENT NUMERALS FOR SAME NUMBERS.</u>														
3010	<u>Expanded notation for whole numbers.</u>								I						8
3015	<u>Expanded notation for fractions.</u>											I			5
3020	<u>Equivalent common fractions.</u>										I				6
3040	<u>Other names for a number.</u>								I						5
3050-3110	<u>ABILITY TO READ AND WRITE.</u>														
3050	<u>Ten basic symbols (0-9).</u>	I													
3070	<u>Whole number units, tens (10-99).</u>							I							

Revision

[illegible]

VOCABULARY • KINDERGARTEN

- | | |
|--------------------|-------------------------|
| 1. curve | 26. member |
| 2. line-segment | 27. ordinal |
| 3. pattern | 28. cardinal |
| 4. triangle | 29. pair |
| 5. rectangle | 30. zero |
| 6. figure | 31. number line |
| 7. numeral | 32. square |
| 8. number | 33. counting |
| 9. shape | 34. numeral |
| 10. estimate | 35. closed |
| 11. measure | 36. circle |
| 12. check | 37. scale |
| 13. empty | 38. pattern |
| 14. join | 39. design |
| 15. more than | 40. square |
| 16. correspondence | 41. separating |
| 17. matching | 42. corner |
| 18. compare | 43. simple closed curve |
| 19. identify | 44. set |
| 20. more than | 45. subset |
| 21. smaller | 46. part |
| 22. larger | |
| 23. inside | |
| 24. outside | |
| 25. largest | |

MATERIALS - KINDERGARTEN

1. individual number line
2. flannelboard and stand
3. tactile numeral-blocks
4. tactile domino blocks
5. walk-on number line
6. stepping stones
7. giant beaded number cards
8. spring scale
9. pictures to illustrate lines, curves, rectangles, squares, triangles, circles
10. set of flannel numbers
11. set of flannel board objects, circle, triangle, square, and rectangle
12. figure and value matching cards
13. number learner
14. bead stair
15. shape matching cubes
16. basic rods and counters
17. Judy clock
18. number group recognition game
19. number trays and plastic counters
20. counting and threading
21. large abacus, individual abacus
22. counting discs, counting sticks
23. domino cards
24. a series of number blocks 1-10
25. number charts- 1-10

IMPORTANT ITEMS

Kindergarten

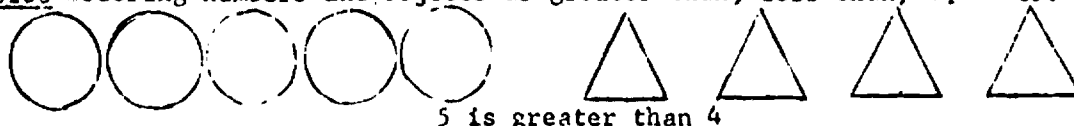
Item	Description	B	M	E
------	-------------	---	---	---

0030	Develop cardinal numbers 1-10.	I		
0060	Counting-cardinal number of set.	I		
4000	Meaning: Set and members (elements).	I		
4010	Equivalent (one-to-one correspondence).	I		
4060 *	Subset			I
3050 *	Ten basic symbols (0-9).	I		
0180 *	Property of one.			I
0090	Backward-rote counting.		I	
0070 *	Ordinal counting-sequence.		I	
0050 *	Number with number line.			I
0040 *	Develop ordinal number sense.			I
0020	Develop cardinal number zero.		I	
4030	Non-equivalent sets.		I	
0150 *	Commutativity-whole number.			I
0120 *	Union of disjoint sets-joining.			I
0100 *	Ordering-equal, greater, less.		I	
4020	Equal sets (identical).		I	
5080	Represent line, point, plane.	I		
5010	Geometric figures in environment.	I		
5030	Relations without measure (size, position).		I	
5040	Two-dimensional figures (plane).		I	
5020 *	Geometric design and pattern.		I	
5060	Curves: simple, closed, open.	I		
9050	Review: numeration.			I

* Difficult Items

METHODOLOGY - KINDERGARTEN

0100 Ordering numbers and objects as greater than; less than; equal to.

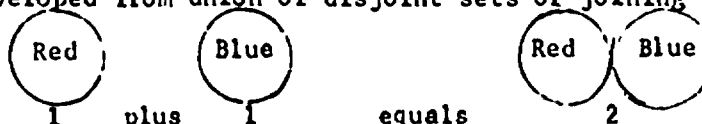


The teacher explains that the number of circles is greater than the number of triangles and that 5 is greater than 4. The children must have the concept of a numeral representing the number of objects in a given collection matching the circles and triangles in one-to-one correspondence, demonstrates that there are more circles than triangles therefore the cardinal number of the set of circles is greater than the cardinal number of the set of triangles.



3 is equal to 3

0120 Addition developed from union of disjoint sets or joining action.



The children should have a basic understanding of sets and elements in a set. Place one cutout on the left side of the flannel board and another (different color or shape) cutout on the other side of the board. Have the children name the number of elements in each set. Have a child join the two sets by moving one set over next to the other set. Note that when two sets of one object each are joined, a new set containing two objects is formed. The sentence illustrates that one plus one and two name the same number. When the two sets are joined, a new set with more members is formed. Introduce the term "addition" as being the mathematical word for combining two numbers, using this joining action of sets for development. Introduce "plus" and "equal", emphasizing that we use these words only with numbers, not with sets.

0130 Addition: Property of one

A set of 1 object will be joined to a set of 3 objects and a new set of 4 objects will be formed.



Bring 3 children to the front of the room ask how many there will be if one more child joins the group. When there are 4 children up front, ask how many there will be if one more is included. Continue in this manner with other examples emphasizing that when we add one to any number our sum is the next number when we are counting.

$$3 + 1 = 4$$

$$4 + 1 = 5$$

$$5 + 1 = 6$$

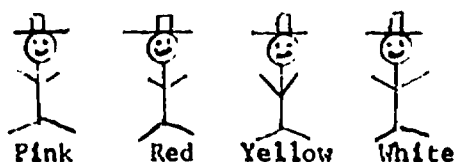
0150 Commutativity, a property of addition

○ ○ ○ ○ ○ ○ ○ ○ ○ $2 + 3 = 5$

○ ○ ○ ○ ○ ○ ○ ○ ○ $3 + 2 = 5$

Have two boys come to the front of the room; then have three more boys join them. Ask how many we have when we join a set of 2 boys and a set of 3 boys or what number $2 + 3$ is. Follow the same procedure, but have the 3 boys up front first and have 2 boys join them. Emphasize that we may join sets or we may add numbers in any order.

0070 Ordinal counting for sequence of numbers.



Place different colored hats on children's heads.

Which child has the pink hat on his head, the red hat, the yellow hat, etc.?

Have the children line up and pretend to be buying tickets. Discuss the position of each child from some starting point and relate to first or second etc. Hence first, second, their etc. are ordinal numbers. Extend the lesson to various series of things in the classroom, for example series of pictures on the bulletin board. Have the children hold cards with the numbers in order and discuss first, second, third, etc.

0500 Associating idea of number with number line. (one-to-one correspondence)



On the board mark two points. Connect the two points to form a part of a line or a line segment. Introduce the phrase "part of a line" has points to show where it ends. Then on a straight line join another "part" and ask how many parts of a line are there now. Draw ten segments. Ask children, "If you were going to use numerals to name the points, what numeral would show the first point with no parts on the line." Label it zero. Then proceed to the next point and ask how many parts of a line are there from zero to the point (mark the point for one with your finger)? Then ask the children to select a number to describe a single part of the line, and continue in this manner.

0040 Developing ordinal number sense.

Line up 3 children in front of the class.



Which child has a hat on his head to make him different? What is the child's position in the line? Ordinal number indicates order or position and tells which one. See 0070, ordinal counting.

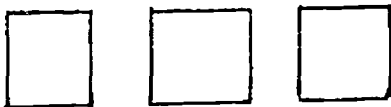
3050 Read and write nonnegative whole numbers for the ten basic symbols (0-9).

Begin with a bare table and ask "How many objects are on the table?" (Accept "none" as an answer.)

The empty set has no members. The set with no members, called the empty set, has a cardinal number, zero.

Place a single object on the table and ask, "How many objects are now in the set?" Introduce the notation card 1 at this time. Continue to place objects on the table, repeat the same quest, and introduce the notation card for other numbers such as 2, 3, etc.

4060 Subsets



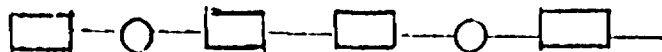
Red blocks



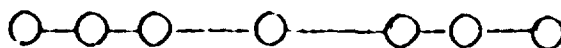
Yellow blocks

Begin by placing the above objects on a table. Explain that even though these blocks have different colors they form a set of five blocks. Remove the yellow blocks. These yellow blocks form a set of two blocks. The yellow blocks form a subset of the set of 5 colored blocks. Similarly, the red blocks form a subset of the set of 5 colored blocks.

5020 Geometric designs or patterns



Using beads of 2 different shapes but of the same color, start a string of beads. Alternate the 2 different shapes. When the string is long enough, point out the sequence of beads - explain that this is a pattern. Have a child continue the pattern. Start another bead pattern and ask which beads should be used next to continue the pattern.



The teacher may have the child make a pattern of his own.

VOCABULARY - GRADE 1

- | | |
|---------------------|--------------------|
| 1. number | 26. order |
| 2. numeral | 27. order rule |
| 3. set | 28. pint |
| 4. fewer | 29. quart |
| 5. greater than | 30. rename |
| 6. less than | 31. abacus |
| 7. add | 32. bundles of ten |
| 8. addends | 33. compare |
| 9. equals | 34. parentheses |
| 10. equations | 35. subset |
| 11. join | 36. subtraction |
| 12. minus | 37. addition |
| 13. missing addends | 38. boundary |
| 14. plus | 39. closed curve |
| 15. separate | 40. curve |
| 16. solve | 41. fold line |
| 17. subtract | 42. fraction |
| 18. sum | 43. line |
| 19. take away | 44. line segment |
| 20. amount | 45. measure |
| 21. contains | 46. outside |
| 22. family of facts | 47. plane |
| 23. grouping | 48. point |
| 24. half-past | 49. rectangle |
| 25. o'clock | 50. region |

51. side
52. unit
53. place value
54. opposite
55. regrouping
56. even
57. odd
58. empty set
59. elements
60. whole number
61. polygon
62. equivalent sets
63. non-equivalent sets
64. inch
65. foot
66. degree
67. match
68. cup
69. one half
70. subset
71. multiplication
72. factors
73. product
74. times

MATERIALS - GRADE 1

1. teacher model abacus
2. number concepts
3. visual fractions
4. time learner
5. Judy clock
6. aluminum liquid measures
7. individual number lines (0-20 or higher)
8. plastic counting rods
9. geometric shapes-squares, triangles etc.
10. individual abacus
11. flannel 3" numbers
12. flannel 7" fractional circles
13. flannel 7" fractional squares
14. count-a-ladder

IMPORTANT ITEMS

Grade 1

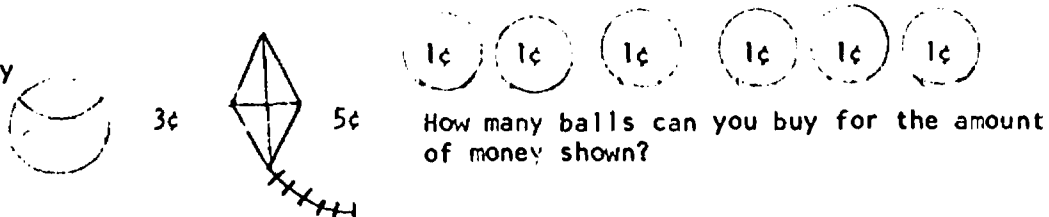
Item	Description	B	M	E
------	-------------	---	---	---

0060	Counting-cardinal number of set.			M
0030	Develop cardinal number 1-10.			M
3050	Ten basic symbols (0-9).			M
0190	Elementary facts of addition.	I		
0130	Addition developed from number line.	I		
0250 *	Subtraction developed from subsets.	I		
0260 *	Subtraction developed from number line.	I		
0310	Elementary facts of subtraction.		I	
0270 *	Inverse of addition-subtraction.		I	
0035 *	Cardinal number beyond 10.		I	
3070	Whole number units, tens (10-99).		I	
0300	Property of one-subtraction.		I	
3080	Units beyond tens.			I
0220 *	More than two addends-no regrouping.		I	
0200 *	Multi-digit addition-no regrouping.		I	
0320	Multi-digit subtraction-no regrouping.		I	
3040	Other names for a number.		I	
0160 *	Associativity of addition.		I	
4040	Unequal sets.		I	
0170 *	Zero-identity element.		I	
0280 *	Property of zero-subtraction.		I	
4050	One-to-many correspondence.		I	
4070	Empty set.	I		
6040 *	Units of measure-money.		I	
3000	Difference between number and numeral.	7		

* Difficult Items

METHODOLOGY - GRADE 1

6060 Money



After the children has a good understanding of prices shown and money values this type exercise could be given. Have the children put the money into sets according to the cost of the item in question then count the number sets. For example, since the item in question is the ball which costs 3 cents, the coins should be put into sets of 3 cents which would give us 2 sets. Therefore, for the amount of money shown 2 balls can be bought. Practice using money in "playstore" situations could prove quite useful in developing background for use of money.

0270 Subtraction is the Inverse of addition.

$$\boxed{} + 3 = 11$$

$$8 + 3 = 11$$

$$11 - 3 = \boxed{}$$

$$11 - 3 = 8$$

Introduce this type of problem using sets of concrete items. For example use 8 girls and 3 boys. Show than when we bring the 3 boys to join the 8 girls we have 11 children. Explain that by subtracting the 3 boys from the set of 11 children we will "undoing" what we have just "done" and that we will end up with the original set of 8 girls. Thus when we know the sum and one of the addends, the missing addend can be found by subtracting the known addend from the sum. The child will know that $8 + 3 = 11$, is one of the basic addition facts he must solve for the missing portion in this problem.

$$\boxed{} + 3 = 11 \text{ or } 8 + \boxed{} = 11$$

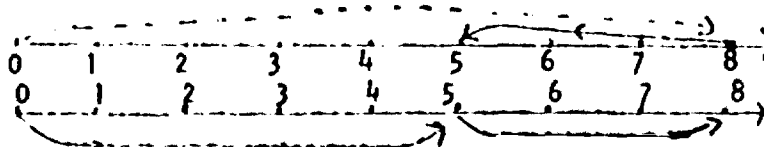
Thus we establish the idea that $\boxed{} + 2 = 5$ is the inverse of $3 + 2 = \triangle$ and that $\boxed{} + 2 = 5$ is the same as $5 - 2 = \boxed{}$

0260 Subtraction from number line.

$$8 - 3 = \boxed{}$$

The teacher should review some basic ideas of addition on a number line such as:

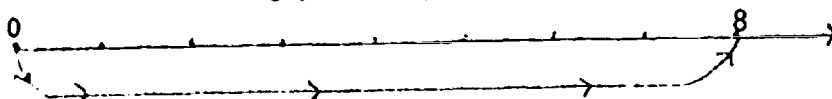
$$5 + 3 = \boxed{}$$



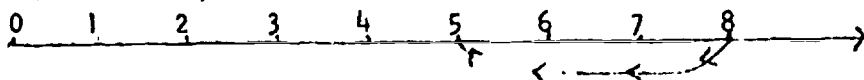
Here the child should note that addition of 2 numbers also implies a direction in traveling along the number line (In this case we moved to the right).

Now we can relate subtraction to the inverse of addition. (See code no. 0270)

For the problem $8 - 3 = \square$, the child must first move 8 spaces from the starting position, zero.



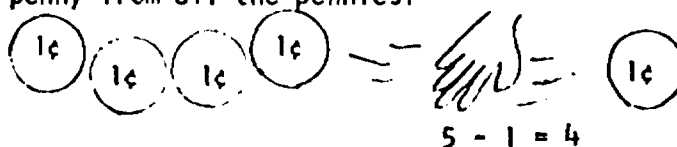
Since subtraction is the inverse of addition, we now move 3 spaces to the left, so that we will subtract three.



Our final stopping position illustrates the end result five. It also can be related to "take away" as used in subtraction. The teacher may use the number line with games, such as "traveling", "taking trips", etc.

0250 Subtraction developed in relation to subsets or separating action.

Separate one penny from all the pennies.



Use sets of concrete items or flannel board objects to show this concept. For example put the five pennies on the desk then "remove" or "separate" one penny from the set and see how many are left of the original set. Continue practice using objects to show separation.

0160 Associativity, a property of addition.



Using objects, demonstrate that it makes no difference in which manner we group the sets together we will end up with the same set of objects. For example if we have 3 red books, 2 blue books, and 4 green books we can group the red and the blue books first then join the green books to that set, or we can group the green and blue books then join the red books to that set. We can relate this grouping of books to the associativity property of addition by concerning ourselves with the number of books in each set. Thus:

$$\begin{array}{ccccccc}
 \begin{array}{|c|c|c|} \hline R & R & R \\ \hline \end{array} & \begin{array}{|c|c|} \hline B & B \\ \hline \end{array} & & \begin{array}{|c|c|c|c|} \hline G & G & G & G \\ \hline \end{array} & \text{or} & \begin{array}{|c|c|c|} \hline R & R & R \\ \hline \end{array} & \begin{array}{|c|c|} \hline B & B \\ \hline \end{array} & \begin{array}{|c|c|c|c|} \hline G & G & G & G \\ \hline \end{array} \\
 (3 + 2) + 4 & = & 3 + (2 + 4) \\
 5 + 4 & = & 3 + 6 \\
 9 & = & 9
 \end{array}$$

0170 Zero, the identity element of addition.

$$\begin{aligned}n + 1 &= 1 + n = n \\4 + 0 &= 0 + 4 = 4\end{aligned}$$

After the children have mastered the concept that 0 is the name given to the empty set, the concept of the identity element can be introduced. Show through the use of objects that regardless of which set we join to an empty set we will have that original set. Show also that if we add the identity element (zero) to any number we will have that number. After these concepts are shown through the use of sets it can also be shown through the use of the number line using zero to denote "not any" steps.

0280 Property of zero in subtraction.

$$\begin{aligned}n - n &= 0 & 8 - 8 &= 0 \\n - 0 &= n & 8 - 0 &= 8\end{aligned}$$

Children should have the understanding that zero represents the empty set and "not any" steps on the number line. If we start at any point, moving "0" steps on the number line keeps us at the same place. See subtraction from number line.

John has the task of dusting the set of books on the top shelf in the library. John cannot reach this particular shelf, so the set of books which he dusted on the top shelf is the empty set. Since the top shelf has eight books in the set and the number of books dusted is zero, if we separate the books dusted from the books on the shelf, there are still eight books to be dusted.

Total books on shelf		Books dusted
{ □ □ □ □ □ □ □ □ }		{ }
8	-	0 = 8

0220 Addition with more than two addends, without regrouping.

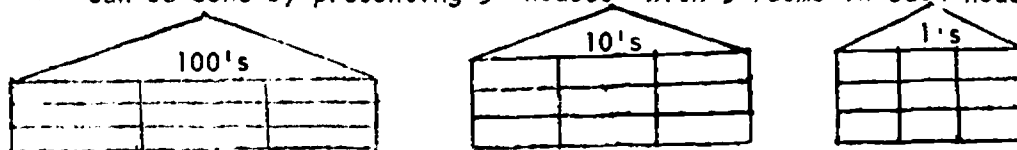
□ □ + △ + ○ ○ ○	= □ □ □ □ □ □
2 + 1 + 3	= 6

$$\begin{array}{r} 2 > \boxed{3} \\ 1 & \\ \hline +3 & \rightarrow +3 \\ \hline 6 & \end{array}$$

In the addition of 3 addends it must be pointed out that first two addends must be added then the third addend added to that sum. For example in the problem $2 + 1 + 4 = 7$ we would first add the 2 and 1 then add the sum 3 to the third addend 4 for a total of 7. The associative property of addition could also be introduced by showing that the 2 and 4 can be added then the 1 added to that sum. The teacher may use objects to illustrate joining the sets two at a time.

0035 Developing cardinal numbers beyond 10.

Develop the idea of place value in the number system. This can be done by presenting 3 "houses" with 9 rooms in each house.

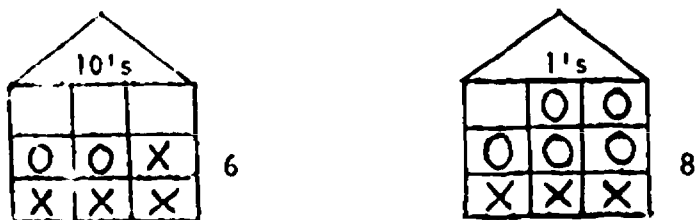


Explain that we can put 1 stick, straw, etc. in each room of Mr. One's place until we get 9 then if we want to add 1 to more we must move the whole "family" of 10 units into 1 apartment of Mr. Ten's place. To find the numeral that represents the number we have, we must count the number of rooms filled in Mr. Ten's place, and the number of rooms in Mr. One's place. Proceed in this manner, moving one more into Mr. One's place and writing the numeral to represent the number until we get to 100. The same procedure may be followed to write numerals in the hundreds. Eventually, the children should then be able to write the numerals without the "houses." During this exercise the numbers should be named in terms of "tens" and "ones." The conventional names can then be presented to the children in decades, (ex. twenties, thirties, etc.).

0200 Multi-digit addition without regrouping.

$$\begin{array}{r} 43 \\ +25 \\ \hline \end{array} \qquad \begin{array}{r} 78 \\ +10 \\ \hline \end{array}$$

After the children know the concept of the tens place and the ones place and understand the concept of joining sets as a basic step for developing the addition, then explain that in the addition of two place addends, we add the ones first then we add the tens. This concept may be developed through the use of the "houses" again.



43 will be 4 x's in the tens house and 3 x's in the one house. 25 will be 2 0's in the tens house and 5 0's in the ones house. Count the number of rooms filled in each house.

Insist that the children add the ones first then add the tens. This is important so that the concept of regrouping may be more easily introduced.

VOCABULARY - GRADE 2

- | | |
|-----------------------|-------------------------|
| 1. add-addend | 26. left |
| 2. missing addend | 27. right |
| 3. cardinal number | 28. separate |
| 4. digit | 29. set |
| 5. doing-undoing | 30. shorter |
| 6. equals | 31. subtract |
| 7. equation | 32. subtraction |
| 8. empty set | 33. minus |
| 9. Family of Facts | 34. take away |
| 10. form | 35. to the left of |
| 11. greater | 36. to the right of |
| 12. greater than | 37. vertical |
| 13. less than | 38. abacus |
| 14. how many | 39. grouped |
| 15. join | 40. grouping |
| 16. less | 41. regrouping |
| 17. longer | 42. number pattern |
| 18. match | 43. one more |
| 19. match one-to-one | 44. order |
| 20. name for a number | 45. pairs |
| 21. number line | 46. paired number lines |
| 22. plus | 47. parentheses |
| 23. sum | 48. pattern |
| 24. numeral | 49. curve |
| 25. remove | 50. end point |

- | | |
|--------------------|-------------------------|
| 51. foot | 76. closed |
| 52. half hour | 77. complex |
| 53. hour | 78. corners |
| 54. line | 79. fraction |
| 55. line segment | 80. gallon |
| 56. measure | 81. inside |
| 57. one half | 82. not closed |
| 58. pint | 83. not simple |
| 59. point | 84. one fourth |
| 60. quart | 85. two fourths |
| 61. standard units | 86. three fourths |
| 62. unit | 87. one third |
| 63. ones | 88. two thirds |
| 64. ones' place | 89. outside |
| 65. set of 10 | 90. rectangle |
| 66. tens' place | 91. region |
| 67. cent | 92. sides |
| 68. dime | 93. simple closed curve |
| 69. half-dollar | 94. square |
| 70. nickel | 95. triangle |
| 71. making change | 96. gro property |
| 72. penny | 97. hur |
| 73. quarter | 98. ren |
| 74. value | 99. ten |
| 75. circle | 100. ol |

- 101. ten tens
- 102. ten ones
- 103. thousand
- 104. value
- 105. equivalent
- 106. even
- 107. odd
- 108. factor
- 109. multiple
- 110. multiplication
- 111. multiply
- 112. product
- 113. times
- 114. one greater than
- 115. whole numbers
- 116. coin

MATERIALS - GRADE 2

1. tactile numeral board
2. folding black flannel board
3. minute marker
4. primary cutouts
5. number line - teacher and individual
6. abacus - teacher model
7. colored counting cubes
8. individual student abacus
9. colored rods
10. counting slide rule
11. time learner
12. visual fractions
13. Judy clock
14. counting disks-500, red
15. analysis of one foot
16. aluminum liquid measures
17. geometric shapes - squares, triangles, etc.
18. count-a'-adder

IMPORTANT ITEMS

Grade 2

Item	Description	B	M	E
3070	Read and write whole number units 10-99.			M
0020	Develop cardinal number zero.			M
4010 *	Equivalent sets (one-to-one correspondence).		M	
4030	Non-equivalent sets.		M	
0210	Multi-digit addition-two addends.		I	
0220	More than two addends-regrouping.			M
0230	More than two addends-regrouping.		I	
0350 *	Multiplication developed from union of equivalent sets.		I	
1140	Addition of fractions-developed from regions.	I		
1090 *	Ordering: greater, less, equal.		I	
0410	Commutativity-multiplication.			I
0570 *	Division-Inverse of multiplication.			I
0540 *	Division developed from equivalent sets.			I
6000	Meaning of measure.	I		
0330 *	Multi-digit subtraction-regrouping.			I
0370 *	Multiplication developed from number line.			I
1020 *	Fractions developed from number line.		I	
5050	Three-dimensional figures.		I	
3100 *	Ability to round number for estimates.			I
5100	Describe and analyze-line, ray.		I	
5090	Describe and analyze-point.		I	
5190 *	Describe and analyze-angles.		I	
3150	Historical systems of numeration.			I
	* Difficult Items			

METHODOLOGY - GRADE 2

0350 Multiplication developed from union of two or more equivalent sets.

$$\begin{array}{c} \text{Concepts: } x \ x \ x \\ 3 \end{array} \quad + \quad \begin{array}{c} x \ x \ x \\ 3 \end{array} \quad = \quad \begin{array}{c} x \ x \ x \ x \ x \ x \\ 6 \end{array}$$

Two sets of 3 are equivalent to one set of 6.
Two three's are 6.

$$2 \times 3 = 6$$

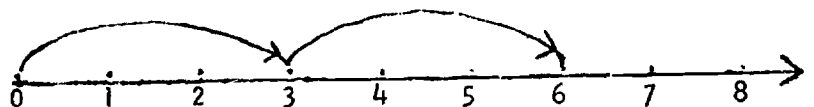
Have 3 girls come to the front of the room to sit at one side of the reading table. 3 more girls come to sit on the other side of the table. Choose a child to tell the number story of the dramatization. Story expected: "Three girls were reading, then three more came. How many girls were reading?" Ask a child to write a number sentence $3 + 3 = 6$. How many 3's did you add? What is two 3's. Write on the board, Two 3's = 6. Further develop the concept with two 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's.

- Activity: 1. Form teams in the following manner:
First, have 2 students on each team, then 3, then 4, and so on. As the teams form, have students name the number of students on each team and on both teams.
2. Chain gang: One student names a number (4) and the next student gives a sum which has two equal addends using that number. The next student names a new number (6) and the next student says twice 6 is 12, and so on. The object is to get around the room without breaking the chain.

0370 Multiplication developed from the number line.

Draw a line on chalkboard. Have a student show the moves for addition of 2 equal addends, beginning at 0 and moving to the right 2 moves. Ex. $3 + 3$ means to move from 0 to 3 to 6. Ask, "How many times did you move?" response, "Twice," write $2 \times 3 = 6$ on the board. Ask a student to explain the \times symbol. Solve the equation, compare with moves on number line. Continue drill to insure that the student understands the connection between multiplication and addition of equal addends.

$$\begin{array}{l} 2 \times 3 = 6 \\ \text{or} \\ 3 + 3 = 6 \end{array}$$



0410 Commutativity, a property of multiplication.

000

000

00 00 00

2 sets of 3 objects is 6 objects

3 sets of 2 objects is 6 objects

two 3's is 6
so $2 \times 3 = 3 \times 2$

three 2's is 6
so $2 \times 3 = 3 \times 2$

The teacher should review the commutative property of addition, (see 0150), to demonstrate that the order in which we add two numbers does not affect the end result. Will the order in which we multiply affect the result? no

To illustrate further, see 0360 - arrays.

3 columns	The two rows have 3 objects each for a
XXX	total of 6 objects.
2 rows xxx	The three columns have 2 objects each for a
	total of 6 objects.

The teacher may devise activities to illustrate other examples.

0540 Division developed from partitioning into equivalent sets.

<table border="1"><tr><td>x</td><td>x</td></tr><tr><td>x</td><td>x</td></tr><tr><td>x</td><td>x</td></tr></table>	x	x	x	x	x	x	How many sets of 3 objects can be made from a set containing 6 objects, or if six objects are separated into two equivalent sets, how large will each set be?
x	x						
x	x						
x	x						

The students should know the relationship between addition and multiplication, multiplication facts in which 2 is the multiplier, and the commutative principle of multiplication.

Arrange 8 books (or other objects) in a row on a table. Have a child dramatize a number story such as: Mary had 8 books. She gave 4 books to each of her friends. How many books did Mary give to her friends? Have Mary take 4 books to a friend, then 4 more to another friend. Then ask, "How many friends got 4 books each?" "How many 4's in 8?" "How many sets of 4 did Mary make?" "How many 4's in 8?"

As the child works with discs, develop more statements as above for sets of 3's in 6, sets of 2's in 4, etc.

Note that we are partitioning a given set into two equivalent sets. The same approach may be applied to three equivalent sets, and others.

0570 Division, the inverse of multiplication.

If $6 \times \square = 12$ then $12 \div 6 = \square$

Review related multiplication facts, division facts with 2 as the divisor, and related division facts with 2 as the quotient.

Use large counting objects for an example. Write on the board how many 6's in 12? How many 2's in 12? Use dramatization to develop this. Then illustrate the relationship of division to multiplication.

$$2 \times 7 = 14$$

$$14 \div 7 = 2$$

0330 Multi-digit subtraction with regrouping.

$$\begin{array}{r} 52 \\ -25 \\ \hline 27 \end{array}$$

5 tens and 2 ones = 4 tens + 12 ones
then $40 + 12$
 $\begin{array}{r} -20 + 5 \\ \hline 20 + 7 = 27 \end{array}$

As a background, use a number line at the board as children use one at their seats. Write on the board, $37 + 6 = \square$. Use number line for solution. Write Inverse equation $43 - 6 = \square$. Find solution. Develop $13 - 6 = 7$, $23 - 6 = 17$, $33 - 6 = 27$, $10 - 4 = 6$, $20 - 4 = 16$, $30 - 4 = 26$.

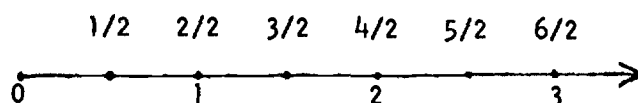
Ask the following questions: What is the sum? What are the addends? How many tens in the sum? How many tens in the missing addend? What is the basic number fact?

Have the children discover that the number of tens in the missing addend is 1 less than the number of tens in the sum. Use abacus or place value chart-rename 1 ten as 10 ones.

$$\begin{aligned} 45 - 6 &= (30 + 15) - 6 && \text{rename} \\ &= 30 + (15 - 6) && \text{regroup} \\ &= 30 + 9 \\ &= 39 \end{aligned}$$

1020 Fractions developed from distances on number line.

Use ruler to develop $\frac{1}{2}$, $\frac{1}{4}$.
Count halves $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$



$\frac{2}{2}$ is another name for 1

$\frac{4}{2}$ is another name for 2

$\frac{6}{2}$ is another name for 3

To reinforce: Have children draw line segments using ruler. Have them label the whole number points or the line then the half way points.

1090 Ordering. Greater than; less than; equal to

Larger than, less than has been taught. By comparing sizes of sets of sticks such as 14 is greater than 12 or $7 + 8$ is greater than 10, 9 is less than 15, $8 + 8$ is equal to 16. To associate the symbol with the words-erase words-substitute the symbol noting that the open area of the symbol is toward the larger numeral and the point toward the smaller numeral.

$$100 < 60 + 50$$

$$4 + 8 > 10$$

$$100 = 50 + 50$$

$$4 + 8 = 12$$

$$100 > 50 + 40$$

$$4 + 8 < 13$$

$$1/2 > 1/3, 1/2 > 1/4, 1/4 < 1/3, 1/4 < 1/2$$

3100 Rounding numbers - estimating answers.

Review sequence charts. Note numerals that are nearest another numeral.

If you were on an elevator at the 27th floor level. Would it be quicker to go to the 30th floor or the 20th floor? Repeat this drill for several examples developing the concept of rounding or estimating numerals.

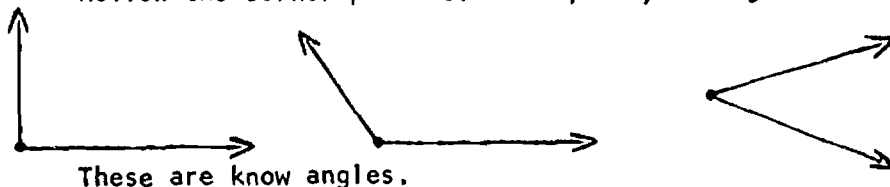
37 rounds to nearest 10 as 40
86 rounds to nearest 10 as 90

Flash cards may be made by each child for 10 to 100. Arrange these cards on each desk so all are in view. Write numerals on chalkboard, ask children to flash the card with the nearest number.

5190 Angles

Concept to be developed: An angle is made up of two rays with a common end point.

Review the corner point of the square, triangle and rectangle.

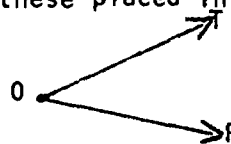


These are know angles.

Procedure. ← Have charts showing these placed in front of room.

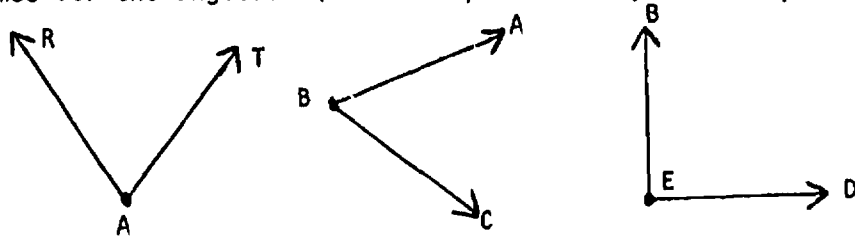


This is an angle.
It is made up of two rays with
a common end point called a
corner point.

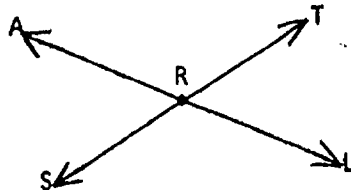


This angle is made up of the two rays.
 \overrightarrow{OT} and \overrightarrow{OP} We may name this angle
as $\angle TOP$ or as $\angle POT$.

Draw various angles, labeling the rays. Emphasize the end point and names for the angles. (note: rays have only one end point)



Note that a number of lines may be drawn through one point.



\overleftrightarrow{AL} and \overleftrightarrow{ST} are lines. They meet at the point R. \overrightarrow{RA} is a ray. Have children name 3 other rays from R. Then have the angles named. ($\angle TRL$ is an angle)

Enrichment activity:

Children may draw rays meeting at a common end point and label as they like. They may list the angles and rays.

VOCABULARY - GRADE 3

- | | |
|-------------------------|-------------------|
| 1. braces | 26. separating |
| 2. cardinal number | 27. angle |
| 3. the empty set | 28. area |
| 4. equivalent | 29. center |
| 5. numeral | 30. circle |
| 6. greater than | 31. curve |
| 7. less than | 32. closed curve |
| 8. ones' place | 33. diagonal |
| 9. tens' place | 34. diameter |
| 10. hundreds' place | 35. end point |
| 11. one-to-one matching | 36. foot |
| 12. place value | 37. half inch |
| 13. set | 38. quarter inch |
| 14. subset | 39. inch |
| 15. total value | 40. length |
| 16. addend | 41. line |
| 17. sum | 42. line segment |
| 18. comparing | 43. measurement |
| 19. difference | 44. parallel |
| 20. remainder | 45. path |
| 21. equation | 46. plane |
| 22. Family of facts | 47. polygons |
| 23. missing addend | 48. quadrilateral |
| 24. opposites | 49. radius |
| 25. remaining | 50. ray |

- | | |
|-----------------------------|----------------------------|
| 51. rectangle | 76. base ten |
| 52. region | 77. common multiple |
| 53. triangle | 78. even number |
| 54. set of points | 79. odd number |
| 55. square | 80. periods |
| 56. square inch | 81. placeholder |
| 57. yard | 82. prime number |
| 58. vertex | 83. finite |
| 59. surface | 84. infinite |
| 60. standard unit | 85. million |
| 61. simple closed curve | 86. multiple |
| 62. digit | 87. quotient |
| 63. inverse operations | 88. cube |
| 64. joining | 89. cylinder |
| 65. division | 90. graph |
| 66. equal addends | 91. lattice |
| 67. factor | 92. parallelogram |
| 68. joining equivalent sets | 93. symmetry |
| 69. multiplication | 94. space |
| 70. product | 95. sphere |
| 71. times | 96. volume |
| 72. expanded numeral | 97. associative property |
| 73. adding equal addends | 98. closure property |
| 74. common factor | 99. commutative property |
| 75. matching sets | 100. distributive property |

101. identity element

102. temperature

103. degrees

104. weight

MATERIALS • GRADE 3

1. folding flannel board
2. felt cutouts of shapes, numerals, and symbols
3. individual number line
4. colored counting cubes
5. abacus
6. visual fractions
7. counting slide rule
8. ABC of solids
9. tactile time teacher
10. individual student abacus
11. tactile numeral board
12. minute minder
13. place value chart
14. hundreds' chart
15. colored rods
16. squared paper
17. yardstick
18. individual rulers
19. geometric shapes
20. sticks for bundles of tens and ones
21. flash cards for +, -, x, ÷
22. pint bottles
23. quart bottles
24. gallon jug
25. individual clocks

IMPORTANT ITEMS

Grade 3

Item	Description	B	H	E
0190	Elementary facts of addition.	M		
0200	Multi-digit addition-no regrouping.		M	
0210	Addition-two addends-regrouping.		M	
0250	Subtraction-developed from subsets.			M
0360	Multiplication-developed from arrays.		I	
0470	Multi-digit multiplication no regrouping.		I	
0480 *	Multi-digit multiplication regrouping.			I
0490 *	Multiplication more than two factors.			I
0510 *	Multiplier or powers of ten as factors.		I	
0575	Distributivity of division over addition.			I
0610	Elementary facts of division.		I	
0620 *	Divisor less than ten-dividend not regrouped.			I
1130	Addition of fractions-developed from number line.		I	
1190	Common fraction notation-equal denominators.		I	
3020 *	Equivalent common fractions.		I	
3090 *	Use of commas to separate periods.		I	
7080 *	Number patterns, rules for divisibility.			I
7090 *	Arithmetic progressions.		I	
0035	Cardinal numbers beyond 10.		M	
0070	Ordinal counting-sequence.		M	
0090 *	Backward-rote counting.			M
0120	Addition-union of disjoint sets.			M
0130	Addition-developed from number line.			M
0260	Subtraction-developed from number line.			M
0450 *	Multiplication-property of zero.		I	
	* Difficult Items			

METHODOLOGY - GRADE 3

0450 Property of zero in multiplication

The teacher may review the commutative property of multiplication for utilization with this topic.

x x x

x x x

 2 threes are 6

x x

x x

x x

 3 twos are 6

or $2 \times 3 = 3 \times 2$

The teacher may begin with 2 boxes such as this.



How many objects are in each box? If the 2 boxes are joined, how many objects will there be? Therefore $2 \times 0 = 0$

Can you write the commutative form of 2×0 ? The child should respond with "the commutative form of 2×0 is 0×2 ". What is 0×2 ?

Illustrate with other similar examples and with this summarizing equation. $2 \times 0 = 0 \times 2 = 0$

It is rather difficult to use a physical world example for 0×2 which means 0 twos, hence the commutative property review may avoid the difficulty.

0090 Other counting, backward, rote, etc.

102, 101, 100, _____, _____

198, 196, _____, _____, 190, _____, _____

90, 91, 92, _____, _____, _____, 96, _____, _____, _____

Review rote counting by use of the number line, pennies and hundreds' chart. In skip counting remove or include progressively as many objects as needed. Children count as they include or remove objects. Since developing the ability to discover patterns is vital to mathematics, let the children investigate a wide variety of patterns on the numeral chart by covering or circling the missing numerals.

7090 Arithmetic progressions

Arithmetic progression is a sequence of numbers each differing from the preceding number by a fixed amount.

3, 6, 9, 12 the constant difference is 3

18, 16, 14, 12 the constant difference is 2

To explain example 1, begin by using the number line and sliding three units on the number line. Follow up by using objects on the flannel board and having children add three progressively. The children may also use geometric shapes such as squares or rectangles at their seat.

If the children discover patterns that aid them, they are alert and should be encouraged. For example adding 9 is the same as adding (10 - 1), or adding 10 and subtracting 1, the tens digit increases from one sum to the next, and the ones' digit decreases.

6, 11, 16, 21, 26, 31 See pattern in ones' place when five is added progressively.

7080 Rules for divisibility

Is 24 evenly divisible by 2? All even numbers can be divided exactly by 2.

It is important that students know what even numbers are. An even number has 2 as a factor. Zero is an even number because it has 2 as one of its factors $2 \times 0 = 0$. All numbers with 0, 2, 4, 6, 8 in the one's place are even numbers. When you count by 2's starting at 0, the numbers you name are even numbers.

When you add any number to itself, the answer is an even number. $3 + 3 = 6$ or 2 sets of threes is 6. Two is a factor of 6.

All numbers represented by numerals ending in 0 or 5 are exactly divisible by five.

How many nickels in 15 cents? Three! How many nickels in 40 cents? Eight! Illustrated that 3 fives = 15 and 8 fives = 40 with the child noticing the numerals 0 and 5 in the unit's or one's place.

If the child has difficulties with money other examples may be used.

3090 Using commas to separate into periods.

By this time the students are familiar that 9 ones and one more one is 1 ten; 9 tens and one more ten is 1 hundred; 9 hundreds and one more hundred is 1 thousand. This may be reviewed by using the abacus, the place value box or chart, and/or the number line.

The thousand place is fourth from the right. You see that in each numeral a comma is used between the three figures at the right and those that stand for thousands.

Period				Period		
H	T	O		H	T	O
	1	6		7	2	6
3	8	2		4	9	3
2	0	4		9	0	6

= 16, 726

= 362, 493

= 204, 906

H means hundreds
T means tens
O means ones

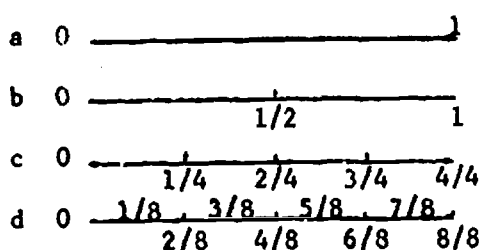
3020 The equivalent common fraction notation.

Place a circular felt cut-out on the flannel board. Cover the region with 2 parts of the same size. Ask what part of the region is being covered by each. Then cover the region with 4 parts of the same size. Ask what part is being covered by each. Remove 2 of the 4 parts and ask what fraction tells how many parts were removed ($2/4$). Then remove the half region and ask the students if the two fourth-regions match the one half-region. Write the numerals $\frac{1}{2}$ and $2/4$ on the board and emphasize that both name the same number.



Several pieces of cardboard of the same size may be used for matching and comparing. Then match the regions 4 eighths with $\frac{1}{2}$, 2 eighths with $\frac{1}{4}$, etc.

The number line is also a good device to develop this concept.



0620 Divisor less than ten, dividend not regrouped, no remainder

$$8 \div 2 = 4 ; 9 \div 3 = 3$$

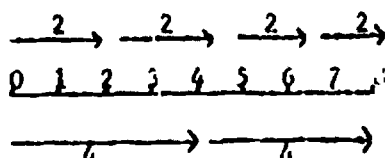
By use of the flannel board or concrete objects review 2 fours = 8, 4 twos = 8, $2 \times 4 = 8$, and $4 \times 2 = 8$. If two 4's equals 8, how many 4's are in 8? If four 2's equals 8, how many 2's are there in 8?

(0 0 0 0) a. $2 \times 4 = 8$ b. How many 4's are there in 8?

(0 0 0 0) c. $4 \times 2 = 8$ d. How many 2's are there in 8?

a. $2 \times 4 = 8$ b. $4 \times 2 = 8$ c. $8 \div 4 = 2$ d. $8 \div 2 = 4$

The sign \div is the division sign. You read the division sentence this way: Eight divided by four is 2. It means that the number of 4's in 8 is 2. This concept may be further developed on the number line.



0510 Powers or multiples of ten as factors.

Begin by reviewing $4 \times 2 = 8$; 4×2 ones = 8 ones. By using bundles of sticks or cards, extend this concept to 4×20 or 4×2 tens = 8 tens or 80. Emphasize the relationship between multiplying ones and multiplying tens.

Exercises such as the following may be written on the chalkboard and illustrated by single sticks and bundles of ten sticks.

$$3 \times 3 = 9 \quad 3 \times 30 = 90 \quad 4 \times 5 = 20 \quad 4 \times 50 = 200$$

The child will also learn that the associative property is useful in justifying multiplication involving multiples of 10. He will learn to approach the problem $3 \times 30 =$ in terms of the base of his system of numeration.

$$\begin{aligned} 3 \times 30 &= 3 \times (3 \times 10) \\ &= (3 \times 3) \times 10 \\ &= 9 \times 10 \\ &= 90 \end{aligned}$$

0490 Multiplication with more than two factors without regrouping.

$$2 \times 1 \times 3 = \square$$

Since the operation of multiplication has the associative property, there are two ways to represent the above product. Consider: $2 \times (1 \times 3)$ = . The 2 represents the number of sets, and the (1×3) represents the number of objects in each of the two sets (one set of three). Using set diagrams:



Two sets of (one set of three). The product $2 \times (1 \times 3)$ represents the total number of objects. Thus $2 \times (1 \times 3) = 6$. Now consider: $(2 \times 1) \times 3 =$.

By the commutative property of multiplication, this can be written $3 \times (2 \times 1) =$. The 3 represents the number of sets, and the (2×1) represents the number of objects in each set (one set of two). Using set diagrams:



Three sets of (one set of two)

The product $3 \times (2 \times 1)$ represents the total number of objects. Therefore, $3 \times (2 \times 1) = 6$ or using the commutative property of multiplication, $(2 \times 1) \times 3 = 6$.

0480 Multi-digit multiplication of two factors with regrouping.

$$\begin{array}{r} 87 \\ \times 3 \\ \hline 21 \\ \underline{240} \\ 261 \end{array}$$

Recall the distributive principle by writing the above in sentence form.

$$\begin{array}{rcl} 3 \times 87 & = & (3 \times 80) + (3 \times 7) \\ & = & 240 + 21 = \boxed{261} \end{array}$$

Let the pupils tell what (3×80) and (3×7) are and let them add the products.

Now write the same example in vertical form as shown in example 1. Get agreement to write the product of the ones first in examples in vertical form. Show that it is easy to multiply first with the figure just above the multiplier and then with the figure in tens' place. Let the pupils discover that the example is like the sentence, except for the form.

**INTERMEDIATE
SECTION**

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0010-0100	<u>BASIC CONCEPTS OF WHOLE NUMBERS.</u>											
0100	Definition: Set of whole numbers.	1			M							
0080	Skip counting.	1			M							
0100	Ordering-equal, greater, less.	K						M				
0110-0200	<u>PROPERTIES OF ADDITION.</u>											
0110	Properties of addition.		I								M	
0140	Closure.		I								M	
0150	Commutativity.	K			E							
0160	Associativity.	1						M				
0170	Zero-the identity element.	1						M				
0230	More than two addends-regrouping.	2			E							
0240-0330	<u>PROPERTIES OF SUBTRACTION-WHOLE NUMBER.</u>											
0240	Definition-binary operation.				I						M	
0270	Inverse of addition.	1						M				
0280	Property of zero.	1									M	
0290	Non-closure, non-commutative, non-associative.										M	
0310	Elementary facts of subtraction.	1				I						
0320	Multi-digit subtraction-no regrouping.	1				M						

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0330	Multi-digit subtraction-regrouping.	2			M							
0340-0520	<u>PROPERTIES OF MULTIPLICATION-WHOLE NUMBER.</u>											
0340	Definition-binary operation.		I					M				
0350	Developed from union of equivalent set.	2	M									
0360	Developed from arrays.	3				M						
0370	Developed from number line.	2			M							
0380	Developed as repeated addition.	1			M							
0390	Developed from product sets.					I						X
0400	Closure.							I				X
0410	Commutativity.	2				M						
0420	Associativity.	3							M			
0430	Distributivity of multiplication over addition.	3							M			
0440	One, the identity element.	3			M							
0450	Property of zero.	3			M							
0460	Elementary facts of multiplication.	1			M							
0470	Multi-digit multiplication, two factors-no regrouping.	3			M							
0480	Multi-digit multiplication, two factors-regrouping.	3				M						
0490	More than two factors-no regrouping.	3			M							

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0500	More than two factors-regrouping.											
0510	Multiples or powers of ten as factors.	3										
0520	Numbers in exponential form.											
0530-0670	PROPERTIES OF DIVISION WHOLE NUMBERS.											
0530	Definition-binary operation.											
0540	Developed from partitioning-equivalent set.	2										
0550	Developed as successive subtraction	3										
0560	Developed from the number line.	3										
0570	Inverse of multiplication.	2										
0575	Distributivity of division.	3										
0580	Property of one.	3										
0590	Zero not a divisor.											
0600	Nonclosure, noncommutative, nonassociative.											
0610	Elementary facts of division. Divisor less than 10.	3										
0620	No regrouping - no remainder. Divisor less than 10.	3										
0630	Dividend not regrouped - remainder. Divisor less than 10.	3										
0640	Dividend regrouped - no remainder. Divisor less than 10.	3										
0650	Dividend regrouped - remainder.											

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0660	Divisor ten or greater.				I					M		
0665	Powers or multiples of 10 as divisors.	3							M			
0670	Numbers expressed in exponential form.								I			I
0680	Solve problems - two operations.	3					M					
0690	Problems - more than two operations.				I							X
0700	Raise numbers to powers - find roots.							I				X
1000-1100	<u>BASIC NUMBER CONCEPTS WITH FRACTIONS.</u>											
1000	Definition - set of nonnegative fractions.						I					X
1010	Developed from subset of a given set.	1					M					
1020	Developed from distances on number line.	2								M		
1030	Developed from plane and solid regions.	1								M		
1040	Whole number related to set of fractions.				I						M	
1050	Renaming from set of fractions.	3								M		
1060	Definition: equality.				I							I
1070	Expressed as terminal or repeating decimal.							I				X
1080	Counting.	3					M					
1090	Ordering: greater, less, equal to.	2								M		
1100	Density.				I							X
1110-1210	<u>PROPERTIES OF ADDITION OF FRACTIONAL NUMBERS.</u>											
1110	Definition: binary operation.						I				M	
1120	Developed from union of disjoint sets.				I		M					
1130	Developed from the number line.	3					M					
1140	Developed from plane or solid regions.	2						M				

II. Items to be introduced

A. Addition with more than two addends, without regrouping

1. Prerequisite behavior

- Identifies the number in each subset and the number altogether for sums up to 12.
- Add two numbers in horizontal or vertical notation form without carrying.
- Adds and subtracts numbers in the tens ending in zero and compares these operations with related elementary facts with numbers less than 10.

2. Behaviors to be taught

- Adds three numbers less than 10 in different ways showing the associative property of addition. Sums to 12
- Adds three numbers less than 10 by adding the numbers in the parentheses first. Sums to 12
- Identifies the number in each of a number of subsets and the number altogether, writes the numerals or responds orally. Numbers to 20
- Solves addition in vertical or horizontal form without pictures. Sums to 20.

3. Enrichment

- Game: Sum Fun

B. Ordering-greater, less, equal.

1. Prerequisite behaviors

- Place or between two numerals to indicate the greater or less for numbers to 12.
- Supplies the correct sign $<$ or $>$ to show the relationship between numbers to 200.
- Labels a shaded figure with appropriate fractional notation for any fraction $\frac{1}{2}$, $\frac{1}{3}$, to $\frac{1}{4}$.

2. Behaviors to be taught

- Provides correct sign $<$ or $>$ to show the relationship between whole objects divided into halves, thirds, fourths with one shaded part.
- Uses three feet in one yard to solve problems of comparison between inches, feet and yards.
- Identifies $\frac{1}{2}$ inch as one of two equal parts making up one inch, 6 objects in $\frac{1}{2}$ dozen; 6 inches as $\frac{1}{2}$ of a foot.

C. Addition of fractions developed from regions

1. Prerequisite behaviors

- Identifies congruent regions and sub-regions.
- Labels a shaded figure with $\frac{1}{2}$, $\frac{1}{3}$, to $\frac{1}{4}$.

2. Behaviors to be taught

- Adding of shaded regions and associating fractional parts with the whole region using $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.
- Extend concepts to include $\frac{1}{2}$.

D. Meaning of measurement (direct, indirect)

1. Prerequisite behaviors

- Constructs and/or uses a number line, 0 to 20, to state what number comes before or after a number or between two numbers. Extend number line to include numbers to 200.

2. Behaviors to be taught
 - a. Identifies the number of units in the length of a line segment.
 - b. Uses a ruler for measuring to the nearest inch.
 - c. Identifies the units of liquid measure, cups, pints, quarts, half gallons and gallons.
- E. Addition with more than two addends, with regrouping.
 1. Prerequisite behaviors
 - a. Associative property of addition is reviewed.
 - b. Solves addition equations in horizontal form sums to 20
 - c. Add two or more addends using addition algorithm for regrouping.
 2. Behaviors to be taught
 - a. Adds with regrouping to sums of 200 with two or more addends using a number line.
 - b. Uses other names for numbers (with or without structured groups) to write alternate statements of a sum.
 - c. Solves one-step word problems in addition.
 - d. Uses addition algorithm to add three digit numbers with regrouping and sums to 1000.
- F. Multiplication developed from union of two or more equivalent sets.
 1. Prerequisite behaviors
 - a. Identifies the cardinal number of a structured group orally and by writing or selecting a numeral.
 - b. Identifies the number in each of a number of subsets and the number there are altogether for sums up to 12.
 2. Behaviors to be taught
 - a. When given pictures of two or more equivalent sets whose sum is 20 or less supplies orally the number altogether.
 - b. Answers questions of the type "How many sets of each are equivalent to a set of ___" by partitioning sets. Products to 25.
 - c. Commutative property of multiplication (Ex. ___two's are ___ or 3 set of 2 are ___.)
 - d. Using the x sign to complete equations using sets of pictures.
 - e. Solves problems represented by objects in arrays and completes multiplication equations with the x sign or repeated addition form. Products to 16.
- G. Fractions developed from distances on the number line.
 1. Prerequisite behaviors
 - a. Same as those used for meaning of measurement.
 2. Behaviors taught
 - a. Compares fractional numbers using the number line; $\frac{1}{2}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}$.
 - b. Locates fractional numbers including improper fractions and negative fractions on a number line. Numerators to 4.
 - c. Measures a line segment to the nearest $\frac{1}{2}$ inch, and $\frac{1}{4}$ inch.
- H. Three-dimensional figures (solid)
 1. Prerequisite behaviors
 - a. Responds correctly to directions which distinguish between a plane geometric shape and the region inside. Identifies, circular, rectangular, triangular, or square regions.
 - b. Counts number of sides, right angles, or angles which a polygon has.
 - c. Identifies a triangle, equilateral triangle, square, rectangle, circle and vertices of a polygon.

2. Behaviors taught
 - a. Selects objects having the same shape or distinguishing features from a collection of objects or pictured objects.
- I. Describe and analyze: point
 1. Prerequisite behaviors
 - a. Points on a number line identified.
 2. Behaviors taught
 - a. Identifies a line as a set of points. (Taught as a part of lines, segments, and rays)
- J. Describe and analyze: angles
 1. Prerequisite behavior
 - a. Identifies a ray as a figure formed by one fixed end point and a line extending indefinitely in one direction.
 2. Behaviors taught
 - a. Identifies an angle as the figure formed by two rays with a common endpoint.
 - b. Count the number of sides, right angles, or angles which a polygon sides to n.
 - c. Identifies a right angle as the angle forming a square corner or as an angle which has 90° .
- K. Describe and analyze: line, line segment, rays.
 1. Prerequisite behaviors
 - a. Identifies a triangle, equilateral triangle, square, rectangle, pentagon, circle and vertices of a polygon.
 2. Behaviors taught
 - a. Identifies a line as a set of points.
 - b. Identifies a line segment as finite in length and part of a line.
 - c. Identifies a line segment, endpoints, a ray as a figure formed by one fixed endpoint and a line extending indefinitely in one direction.
 - d. Identifies a representation for a line as a straight line with arrowheads on each end, showing that the line goes on without stop.
 - e. Identifies an angle as the figure formed by two rays with a common endpoint.
 - f. Names a line by letters of any two points on it.
- L. Multi-digit subtraction-regrouping
 1. Prerequisite behaviors
 - a. Adds two numbers both in the tens, or one in the hundreds and one in the tens for sums to 200 using expanded notation.
 - b. Adds two or more addends (to 3 digits) using the addition algorithm for regrouping.
 - c. Completes subtraction problems with minuends to 200 by using strategy groups.
 - d. Subtracts numbers in horizontal or vertical form without regrouping. Numbers to 200.
 2. Behaviors to be taught
 - a. Completes subtraction problems with minuends to 200 by using a number line.
 - b. Performs repeated subtraction with minuends to 200 and subtrahends or any multiple of 10.
 - c. Subtracts using expanded notation and regrouping, also using the subtraction algorithm in vertical numbers to 200.
 - d. Performs subtraction for problems with numbers to 3 digits using above method.

M. Commutatively, a property of multiplication (whole numbers)

1. Prerequisite behaviors

- a. Illustrates the use of commutative property of addition by writing two different addition equations for the same pictured situation. (to 12)
- b. Completes 2 multiplication equations written as "___ two's are ___" or "3 sets of 2 are ___" for the same set of pictured objects which show the commutative property. Products to 12.

2. Behaviors to be taught

- a. Uses pictured arrays to illustrate and/or solve simple problems involving the commutative property for multiplication.
- b. Completes two multiplication equations written with the \times sign which are represented by the same set of pictured objects and together show use of the commutative property for multiplication. Products to 12. (with or without pictures)

N. Division-developed from partitioning into equivalent sets.

1. Prerequisite behaviors

- a. When given pictures of two or more equivalent sets whose sum is 20 or less, supplies orally the number altogether when there are ___ sets of ___ each.
- b. Answers questions of the type: "How many sets of ___ each are equivalent to a set of ___" by partitioning sets. For products to 25.

2. Behaviors to be taught

- a. Divides a set of up to 12 members into a requested number of equivalent sets and states the number in each of the sets and the number of remaining objects, if any.
- b. Writes the missing factor and quotient for a pictured array showing that multiplication and division are inverses. Products to 20.
- c. Demonstrates with objects or pictured arrays that $a/b=x$ means that we can divide a to make x sets with b in each set or that if we have b sets there will be x things in each set.

O. Division-inverse of multiplication

1. Prerequisite behaviors

- a. Same as above for equivalent sets in division.
- b. Divides a set of up to 12 members into a requested number in each of the sets and the number of remaining objects, if any.

2. Behaviors to be taught

- a. Writes or completes the related division problem for a multiplication problem with product 10 or less.

P. Multiplication developed from the number line.

1. Prerequisite behaviors

- a. Find the sum of two or more addends by using the number line
- b. Adds with regrouping to sums of 200 with two or more addends using a number line.
- c. Writes the answers when both the repeated addition form and the multiplication form written with the \times sign are presented. Products to 200.

2. Behaviors to be taught

- a. Uses a number line monogram to solve multiplication problem for facts through 10×10 .
- b. Solves problems (multiplication) for products to 200 using the number line.

- Q. Multi-digit addition two addends with regrouping.
1. Prerequisite behaviors
 - a. Writes a numeral as hundreds, tens, ones.
 - b. Adds two numbers in horizontal or vertical notation form without carrying.
 2. Behaviors to be taught
 - a. Adds with regrouping to sums of 200 with two or more addends using a number line.
 - b. Adds all numbers in the tens to sums of 100 (with regrouping) using the addition algorithm.
 - c. Adds two or more addends (to 3 digits) using the addition algorithm for regrouping.
- R. Rounding Numbers
1. Prerequisite behaviors
 - a. Taught previously to this
 2. Behaviors to be taught
 - a. Finding the nearest numeral, ten using the number line.
 - b. Using the number line to find the nearest hundred.
- S. Historical systems of numeration
1. Prerequisite behaviors
 - a. Writes numerals, identifies sets, identifies place value of the units and tens digit in numbers 10 to 20.
 - b. Identifies cardinal number of a structured group.
 2. Behaviors to be taught
 - a. Identify and write Roman numerals associated with Arabic numerals.
 - b. Identifies the Egyptian and Babylonian numeral systems orally.
 - c. Associated Roman Numerals with the Arabic numerals on the clock face.
 - d. Identifies book volumes labeled with Roman numerals.

III. Optional items

- A. Ordinal numbers first to twentieth
1. Calendar - through thirty first
 - a. days, months
- B. Measure
1. Time - hour, half hour, five minute intervals - quarter to, and quarter after.
- C. Money
1. Nickel, dime, quarter, half dollar and dollar.
- D. Secret code puzzles - addition and subtraction drills.
- E. Cross Number Puzzles
- F. Open sentences in all number operations.

IV. Enrichment activities

- A. A file of additional activities for enrichment is available. Games, directed mathematics skills, and activities are suggested.

Third Grade
Primes Math

I. Items to be mastered

A. Elementary facts of addition

1. Prerequisites

- a. Illustrates the use of the commutative property of addition by writing two different addition equations for the same pictured situation.
- b. Finds the sum of two or more addends by using the number line.
- c. Completes problems in which zero is one of the addends.

2. Behaviors to be taught

- a. Finds other names for numbers to 20 by matching given addition expressions.
- b. Finds other names for numbers to 20 by writing or completing simple addition expressions.
- c. Solves one-step word problems with pictures which require the use of addition facts with numbers to 20.
- d. Solves addition problems written in vertical notation form without pictures. Sums to 20
- e. Constructs and/or completes an addition table and uses the table to solve addition problems.

3. Enrichment

- a. Tape-Riddles

B. Cardinal numbers beyond ten

1. Prerequisites

- a. Adds to a second set or draws a second set of objects and identifies one set as having more, fewer, or the same number of objects as the other set.
- b. Identifies the cardinal number of a structured group. Responds by writing or selecting a numeral.
- c. Writes the proper numeral when shown a set written in notation followed by an equals sign.

2. Behaviors to be taught

- a. Identifies the number of tens and ones in pictures representing numbers to 200 by writing or selecting the correct numeral.
- b. Renames or regroups numbers according to the place value of the digits - e.g., 275 = 2 hundreds, 7 tens, 5 ones, or 2 hundreds, 5 tens, 5 ones.
- c. Abstracts the cardinal number from structured groups to 1000 in number. Writes the standard numeral. Objects are grouped into bundles of hundreds, tens, ones.

C. Ordinal counting-sequence

1. Prerequisites

- a. Identifies how many more or less one set has when given two nonequivalent sets.
- b. Supplies the correct sign $<$ or $>$ to show the relationship between two numbers to 200.
- c. Constructs and uses a number line to 200 to answer questions, such as, what number comes before, after, or between two numbers.

2. Behaviors to be taught
 - a. Locates the requested ordinal position, first to twelfth, in an ordered set of objects.
 - b. Writes the numeral that comes after, before, or between two given numbers to 1,000 to 10,000.
 - c. Writes the missing numerals when presented with part of the sequence of numbers from 1 to 10,000 with blanks or frames for missing numbers.
 - d. Writes the correct numeral for a number given in words up to millions.
 - e. Reads numbers to millions.

D. Multi-digit addition-no regrouping

1. Prerequisites
 - a. Identifies or completes problems which illustrate the commutative or associative property of addition.
 - b. Rewrites a given problem to illustrate the commutative or associative property of addition.
 - c. Writes a numeral in expanded notation form; its "hundreds" part, its "tens" part, and its "ones" part. Renames numbers to 1,000.
2. Behaviors to be taught
 - a. Adds two numbers in horizontal or vertical form without carrying. Sums to 200.
 - b. Supplies or completes an addition and subtraction equation for word problems. Solves the equation.
 - c. Adds using cent notation and decimal notation for money problems to \$2.00.
 - d. Adds two, three or more addends (to five digits) for problems written in horizontal or vertical notation form.
 - e. Adds or subtracts money values using cent notation or decimal notation, no limit to money value.
3. Enrichment
 - a. Tape-find the hidden picture

E. Backward-rota counting

1. Prerequisites
 - a. Abstracts the cardinal number from structured groups to 1000 in number. Writes the standard numeral. Objects are grouped into bundles of hundreds, tens, ones.
 - b. Writes the missing numerals when presented with part of the sequence of numbers from 1 to 10,000 with blanks or frames for missing numbers.
2. Behaviors to be taught
 - a. Writes the numeral for a number that comes after, before, or between two given numbers to 1,000 to 10,000.
 - b. Constructs and uses a number line to 200 to answer questions, such as, what number comes before, after, or between two numbers.

F. Addition developed from union of disjoint set or joining action

1. Prerequisites
 - a. Identifies the cardinal number of a structured group to 12. Responds by writing or selecting a numeral.
 - b. Abstracts the cardinal number from structured groups at 1000 in number. Writes the number as —tens. Objects are grouped into bundles of hundreds, tens, ones.

2. Behaviors to be taught
 - a. Supplies the missing operational sign, +, -, to complete problems for numbers to 100.
 - b. Writes an addition equation using the + sign or a subtraction equation using the - sign to show that the given pictures depict an addition or subtraction equation.
 - c. Identifies or completes problems which illustrate the commutative or associative property of addition.
- G. Addition developed from the number line
1. Prerequisites
 - a. Supplies the correct sign $<$ or $>$ to show the relationship between two numbers.
 - b. Skip counts from any starting point forward or backward by 10's, 5's, 4's, 3's, 2's, 100's, and 50's. Responds by writing numerals to continue the pattern.
 2. Behaviors to be taught
 - a. Finds the sum of two or more addends by using the number line.
 - b. Supplies the missing addend for problems with up to three addends.
 - c. Adds with regrouping to sums of 200 with two or more addends using a number line.
- H. Subtraction developed in relation to subsets or separating action.
1. Prerequisites
 - a. Identifies a subset of a given set by choosing among a number of sets, one or more which would qualify as subsets of the given set.
 - b. Identifies how many more or less one set has when given two nonequivalent sets.
 - c. Matches objects in pictured sets by drawing connecting lines and responds that one set has more, has less, or has the same number as a given set.
 2. Behaviors to be taught
 - a. Takes away enough members of one set to match the numerical value of a given set. Answers: "How many are left?" and "How many were in the first set before?"
 - b. Finds the remainder or a missing term for subtraction exercises.
 - c. Provides or completes a subtraction statement using the - sign and = sign for pictures composed of sets of objects. Responds by writing or selecting the numerals.
 3. Enrichment
 - a. Riddles - General review - no multiplication or division.
- I. Subtraction developed from the number line
1. Prerequisites
 - a. Supplies the missing addend for problems with up to three addends and sums to 25.
 - b. Skip counts from any starting point forward or backwards by 10's, 4's, 3's, 2's, 100's, and 50's. Responds by writing numerals to continue the pattern.
 2. Behaviors to be taught
 - a. Completes subtraction problems with minuends to 200 by using a number line.
 - b. Supplies the missing addend for problems with up to three addends and sums to 25.
 - c. Supplies or completes the inverse problem in subtraction for a given addition problem.

J. Multi-digit addition with regrouping

1. Prerequisites
 - a. Rewrites a given problem to illustrate the commutative or associative property.
 - b. Writes a numeral in expanded notation form, its "hundreds" part, its "tens" part and its "ones" part (renaming numbers). Numbers to 1000
 - c. Supplies an example of a word problem when requested to illustrate addition.
2. Behaviors to be taught
 - a. Adds two or more addends (to five digits) using the addition algorithm for regrouping.
 - b. Solves for the missing addend for problems with numbers to five digits and requiring regrouping.
 - c. Solves one-step word problems in addition.
 - d. Checks answers in addition by applying the commutative property.
3. Enrichment
 - a. Find the hidden picture.

II. Items to be introduced

A. Number patterns - Rules for divisibility

1. Prerequisites
 - a. Solves for the missing addend for problems with numbers to five digits and requiring regrouping. A placeholder may be used for one or more missing digits.
 - b. Subtracts using expanded notation and regrouping for problems involving numbers to 200.
 - c. Finds the missing factor (divisor, quotient) or dividend in division problems with or without placeholders. Numbers to 200.
2. Behaviors to be taught
 - a. Identifies odd and even numbers from 0 to 200 by selecting or writing the numerals or responding orally.
 - b. Writes numerals to continue an addition sequence after discovering the rule intuitively.
 - c. Constructs and uses number families from basic addition facts. Sums to 24
 - d. Writes numbers to continue a subtraction sequence after discovering the rule intuitively.
 - e. Completes "Cross number puzzles" involving subtraction for minuends to 200 (e.g. the difference of the differences of the rows is equal to the differences of the differences of the columns).
3. Enrichment
 - a. Grids - Fill in squares so that all columns and rows add up to a given number.

B. Arithmetic Progressions

1. Prerequisites
 - a. Adds with regrouping to sums to 200 with two or more addends using the addition algorithm.
 - b. Completes subtraction problems with minuends to 200 by using the subtraction algorithm for regrouping.
2. Behaviors to be taught
 - a. See number patterns

C. Common fraction notation - equal denominators

1. Prerequisites
 - a. Writes the number of equivalent subsets there are when a group of objects divided into halves, thirds, fourths, to twelfths.

- C. Common fraction notation - equal denominators
1. Prerequisites
 - a. Writes the number of equivalent subsets there are when a group of objects is divided into halves, thirds, fourths, to twelfths.
 2. Behaviors to be taught
 - a. Identifies $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of regular whole objects by selecting or writing the fraction which describes the shaded object.
 - b. Uses objects, pictured objects, or no pictured objects to identify different fractional statements which are equivalent to each other.
- D. Equivalent common fractions
1. Prerequisites
 - a. Writes the number of parts there are when an object is divided into halves, thirds, fourths, to twelfths.
 - b. Writes the number of equivalent subsets there are when a group of objects is divided into halves, thirds, fourths to twelfths.
 2. Behaviors to be taught
 - a. Behaviors to be taught provides the correct sign $>$, $<$ or $=$ to show the relationship between whole objects divided into halves, thirds, fourths, thirds with one part shaded.
 - b. Uses objects, pictured objects, or no pictured objects to identify different fractional statements which are equivalent to each other.
- E. Addition of fractions - developed from the number line
1. Prerequisite

Uses the number line to identify different fractional statements which have the same numerical value. Fractions to twelfths with numerators to 4.
 2. Behavior to be taught

Locates fractional numbers including improper fractions and negative fractions on a number line. Numerators to four.
- F. Use of commas to separate into periods
1. Prerequisites
 - a. Writes the standard numeral for a number represented as ____thousands, ____hundreds, ____tens, ____ones. Numbers to 10,000.
 - b. Renames or regroups numbers according to the place value of the digits, e.g. 275=2 hundreds, 7 tens, 5 ones; or 1 hundred ____tens, 5 ones; or 2 hundreds, 5 tens, ____ones.
 - c. Writes the standard numeral for a number represented as ____hundreds, ____tens, ____ones. Numbers to 1000.
 2. Behaviors to be taught
 - a. Writes a number in expanded notation form; its "thousands" part + its "tens" part + its "ones" part. Numbers to 10,000.
 - b. Reads numbers to millions
 - c. Identifies the place value of digits in numbers up to millions.
- G. Multiplication developed from arrays
1. Prerequisite
 - a. Identifies the correct objects in a picture when asked to "Find the picture of a set of ____" when a common property is named.
 2. Behaviors to be taught
 - a. Solves multiplication problems represented by objects in arrays and completes multiplication equations written with the x sign or in repeated addition form.

- b. Solves a more difficult multiplication problem by using the results of two smaller arrays to obtain the product of the larger array.
 - c. Checks multiplication problems by reversing the order of the factors and multiplying.
 - d. Constructs or completes a multiplication table through the product 12×12 .
- H. Property of zero in multiplication
 1. Prerequisites
 - a. Uses pictured arrays to illustrate and/or solve simple problems involving the commutative property for multiplication. Products to 18.
 - b. Writes the answers when both the repeated addition form and the multiplication form written with the \times sign are presented.
 2. Behaviors to be taught
 - a. Uses a number line nomogram to solve multiplication problems for facts through 10×10 .
 - b. Gives zero as the product for a multiplication problem for which zero is one of the factors.
- I. Multi-digit multiplication without regrouping
 1. Prerequisites
 - a. Identifies one as the identity element for multiplication.
 - b. Multiplies more than two numbers using the associative principle for multiplication.
 - c. Identifies or selects from a list those number statements which illustrate a certain principle, e.g., associative commutative, distributive, etc.
 2. Behaviors to be taught
 - a. Uses the distributive principle to simplify multiplication of one and two digit factors, e.g., $7 \times 85 = (7 \times 80) + (7 \times 5)$.
 - b. Multiplies a one digit factor times a three or more digit factor.
 - c. Solves one or two-step word problems in multiplication.
 3. Enrichment
 - a. Multiplication puzzle
- J. Multi-digit multiplication with regrouping
 1. Prerequisites
 - a. Uses the associative principle to simplify multiplication of one and two digit factors; e.g., $2 \times (98 \times 50) = (2 \times 50) \times 98$.
 2. Behaviors to be taught
 - a. Solves multiplication problems for facts 10×10 using the multiplication algorithm for problems written in vertical or horizontal form.
 - b. Multiplies a one digit factor times a three or more digit factor. Uses algorithm with or without regrouping in tens, hundreds, or thousands place.
- K. Multiplication - more than two factors.
 1. Prerequisites
 - a. Solves multiplication problems by using other names for numbers (renaming) and the distributive property.
 2. Behaviors to be taught
 - a. Multiplies more than two numbers using the associative principle for multiplication.
 - b. Uses the associative principle to simplify multiplication of one and two digit factors.
- L. Multiplier or powers of ten as factors
 1. Prerequisites
 - a. Completes two multiplication equations which together illustrate use of the commutative property for multiplication. Pictures may or may not be presented with the problem.

2. Behaviors to be taught
 - a. Uses multiples of ten to generalize multiplication and division facts, e.g. $3 \times 9 = \underline{\quad}$, $3 \times 90 = \underline{\quad}$, and $3 \times 900 = \underline{\quad}$.
 - b. Multiplies using the algorithm for multiples of powers of 10, e.g. $8 \times 10 = 80$, $8 \times 20 = 160$, $8 \times 100 = 800$.
 - c. Uses the division algorithm to solve problems for multiples of powers of 10, $100 \div 4$, $20 \div 4$.

M. Distributivity of division over addition

1. Prerequisites
 - a. Divides a set of up to 18 members into a requested number of equivalent sets and states the number in each of the sets and the number of remaining objects, if any.
 - b. Supplies or completes an inverse problem in division for a given multiplication problem; with or without remainders.
 - c. Supplies or completes inverse problem in multiplication for a given division problem.
2. Behaviors to be taught
 - a. Illustrates that division is distributive over addition and subtraction when given a dividend and a divisor.
 - b. Uses the distributive principle to simplify division problems for two or three digit products; e.g. $248 \div 8 = (240 + 8) \div 8 = (240 \div 8) + (8 \div 8)$
 - c. Varies the placement of parenthesis in a problem to generate different problems and then solves each.
 - d. Identifies or selects from a list those number statements which illustrate a certain principle; e.g. Associative, commutative, distributive, etc.

N. Elementary facts of division

1. Prerequisites
 - a. Solves division problems by performing repeated subtractions.
 - b. Writes or completes the related division problem for a multiplication problem with product 18 or less.
2. Behaviors to be taught
 - a. Writes division equations when presented with written stories.
 - b. Solves division problems for dividends to 100 with whole number quotients by using the division algorithm.

O. Division - divisor less than ten, dividend not regrouped, no remainder.

1. Prerequisites
 - a. Identifies one as the identity element for division.
 - b. Checks answers in addition, subtraction, multiplication, division by using the inverse operation.
2. Behaviors to be taught
 - a. Divides money values by repeatedly subtracting multiples of the divisor using cent notation or decimal notation to dividends to 200; with or without remainders.
 - b. Divides a three or more digit product by a one digit factor by using the algorithm with regrouping or without regrouping. Checks division problem by multiplication.

III. Optional

- A. Multi-digit subtraction regrouping (This is an important item although not mastered or introduced at this grade level).
- B. Ability to round numbers for estimates.
- C. Money
- D. Time
- E. Temperature
- F. Tables, charts, graphs.
- G. Describe and analyze geometric figures.

Grade Four
Primes Math

I. Items to be mastered

A. Elementary facts of subtraction

1. Prerequisites

- a. Constructs and completes an addition and uses the table.
- b. Solves addition problems.
- c. Fills in frames for missing addend problems.
- d. Completes the inverse problem in addition.

2. Behaviors to be taught

- a. Using the number line, finds the remainder of a missing term for subtraction.
- b. Completes addition and related subtraction equations.
- c. Finds the missing terms for a collection of addition and subtraction problems.

3. Enrichment

- a. Enrichment -- dots and squares

B. Multiplication developed from equivalent sets

1. Prerequisites

- a. Abstracts the cardinal number from pictures of structured groups to five in number. Responds by writing a numeral.
- b. Matches objects in pictured sets by drawing connecting lines and responds that one set has more, has less, or had the same number as a given set. Sets to five.
- c. Identifies a subject of a given set by choosing among a number of sets, or more which would qualify as subsets of the given set.

2. Behaviors to be taught

- a. Answers questions of type "How many sets of each are equivalent to set o
- b. Completes 2 multiplication equations written as twos are on 3 sets of 2 are for the same set of pictured objects which show the use of the commutative property. Products to 18.
- c. Completes two multiplication equations written with X sign which are represented by the same set of pictured objects and together show use of the commutative property for multiplication. Products to 18.
- d. Completes or writes problems written in terms such as twos are 3 sets of 2 are, with the word times, in equation form with the x sign or in repeated addition form. Products to 200.

3. Enrichment

- a. None

C. Multi-digit subtraction no regrouping

1. Prerequisite

- a. All subtraction combinations
- b. Completes cross no. puzzles involving addition to sums to 200.
- c. Rounds numbers to nearest thousand, hundred, thousand or million for estimation of answers for problems.

2. Behaviors to be taught

- a. Performs subtraction with numbers to 5 digits and not requiring regroup
- b. Constructs and uses number families for solutions in subtraction to 10,

D. Addition with more than two addends with regrouping

1. Prerequisite
 - a. Rewrites a given problem to illustrate this commutative or associative property of addition.
 - b. Adds two, three or more addends — no regrouping
 - c. Adds two, three or more addends with regrouping.
 - d. Adds 3 or more addends using the commutative principle or associative principle.
2. Behaviors to be taught
 - a. Adds or subtracts money values.
 - b. Solves one step —word problems involving combination of any arithmetic operations.
3. Enrichment
 - a. Cross number puzzle.

E. Commutativity, a property of

1. Prerequisite
 - a. Finds the missing terms for a collection of addition and subtraction problems written in horizontal or vertical form. Sums to 20. Missing terms maybe represented by a letter.
 - b. Constructs and/or completes an addition table.
2. Behaviors to be taught
 - a. Identifies or completes problems which illustrate the commutative or associative property of addition.
3. Enrichment
 - a. None

F. Multiplication developed from number line.

1. Prerequisite
 - a. Skip counting
 - b. Adds two numbers both in the tens
 - c. Adds 200 more addends to 5 digits using the additve algorithm for reg
2. Behaviors to be taught
 - a. Solves multiplication problems represented by object in arrays and com multiplication equations written with the x sign or in repeated additi form. Products to 200.
 - b. Solves multiplication problems through products to 200 using repeated addition.

G. Subtraction — Inverse of addition

1. Prerequisites
 - a. Adds and subtracts numbers in the hundreds ending in zero, with or without structured groups and compares these operations with the relat elementary fact with numbers less than 10.
 - b. Does problem relating to the closure property of addition.
 - c. Writes numbers to continue a subtraction sequence after discovering th rule intuitively.
2. Behaviors to be taught
 - a. Supplies or completes the inverse problem in addition for a given subtraction problem.
 - b. Supplies or completes the inverse problem in subtraction for a given addition problem.
 - c. Checks subtraction problems by the difference and subtrahend to equal the minuend.

H. Multi-digit subtraction with regrouping

1. Prerequisites

- a. Performs subtraction for problems with numbers to 5 digits — no regrouping
- b. Constructs and use number families for solutions in subtraction minuends to 10,000.
- c. Writes numerals in expanded form.
- d. Adds two numbers both in tens, or one in hundreds and one in tens for sums to 200 (regrouping or renaming)

2. Behaviors to be taught

- a. Using number line to complete subtraction with minuends to 200.
- b. Subtracts using cent notation or decimal notation for money problems.
- c. Subtraction for problems of 5 digits — (regrouping)

I. Multiplication developed as repeated addition

1. Prerequisites

- a. Behavioral analysis is progress

2. Behaviors to be taught

- a. Solves multiplication problems through products to 200 using repeated addition.
- b. Using arrays to solve multiplication problems.

3. Enrichment

- a. Game — Build you Thinking Power.

J. Elementary facts of multiplication

1. Prerequisites

- a. Answers questions of the type "how many sets of each are equivalent to a set of M" by drawing pictures of sets or arrays.
- b. Solves multiplication problems through products to 200 — using repeated addition.
- c. Completes 2 multiplication equations which together illustrate use of commutative property for multiplication. Products to 100.

2. Behaviors to be taught

- a. Checks multiplication problems by reversing the order of the factors and multiplying.
- b. Constructs or completes a multiplication table through the product 12×12 .
- c. Solves multiplication problems by using other names for numbers (renaming) and the distributive property. Products to 200.

3. Enrichment

- a. Puzzle

K. More than 2 factors — no regrouping

1. Prerequisites

- a. Completes 2 multiplication equations which together illustrate use of the commutative property for multiplication.
- b. Identifies the product and/or factors of a problem. Products to 200.

2. Behaviors to be taught

- a. Constructs and completed a multiplication through the product of 2×12 .

L. Elementary facts of division

1. Prerequisites

- a. Writes or completes the related division problems for a multiplication problem with product 18 or less.
- b. Checks multiplication problems by dividing the product by one of the factors to get the other factor.

2. Behaviors to be taught
 - a. Constructs a rectangular array to illustrate a given divisor statement.
 - b. Writes or completes the related division problem for a multiplication problem with product 18 or less.
 - c. Solves multiplication and division problems with small whole numbers solutions with placeholders for missing numbers by performing the inverse operation to the one indicated. Numbers to 200.

M. Division developed from equivalent sets.

1. Prerequisites
 - a. Matches objects in pictured sets by drawing connecting lines and responds that one set is more, has less, or has same number as a given set.
 - b. Answers questions of the type how many sets of each are equivalent to a set of M.
 - c. Identifies a subset of a given set by choosing among a number of sets, one or more which would qualify as subsets of the given set.
2. Behaviors to be taught
 - a. Solves division problems by repeatedly subtracting multiples of the divisor. Dividends to 200 with remainders.
 - b. Solves division problems by performing subtraction of the sets.

N. Divisors less than 10 - no regrouping no remainder

1. Prerequisites
 - a. Solves multiplication problems for facts through 10×10 - using the multiplication algorithm for problems written vertical or horizontal.
 - b. Multiplies a one digit factor times a 3 or more digit factor.
 - c. Uses the distributive principle to simplify multiplication of one or 2 digit factors.
2. Behaviors to be taught
 - a. Find the missing factor in multiplication with missing factors to 10 and products to 100.
 - b. Finds the missing factor or dividend in division facts, $5 \times 9 = 3 \times 90 = 3 \times 90$.
 - c. Divides a 3 or more digit product by a one digit factor by using the algorithm.

O. Definition: set of whole numbers

1. No prerequisites
2. Behaviors to be taught
 - a. Identifies the cardinal number of a structured group to 12.
 - b. Identifies a set of whole numbers as a set beginning with zero as the smallest number and having no greatest whole number.
3. Enrichment: Using a code

II. Items to be introduced

A. Whole numbers related to a set of fractions

1. Prerequisites
 - a. Shows by example that the numerator of a fraction means that many out of a number of equal parts expressed by the denominator for the fractions, $\frac{2}{2}$, $\frac{2}{3}$, $\frac{2}{4}$.
 - b. Identifies other names for numbers by choosing from a selection of proper fractions, improper fractions, mixed fractions.

2. Behaviors to be taught
 - a. Supplies correct sign $>$ $<$ or $=$ to show comparisons of fractions.
 - b. Uses objects, pictured objects to identify different fractional statements which are equivalent to each other.
 - c. Finds a set of equivalent fractions for a given fraction using higher and lower terms.
- B. Closure, a property of addition
 1. Prerequisites
 - a. Does problems relating to closure of addition
 2. Behaviors to be taught
 - a. Identifies which operations or number system (whole numbers) have the closure property.
 3. Enrichment - Writing numerals to hundred thousands.
- C. Subtraction - Non closure, non commutative, non associative.
 1. Prerequisites
 - a. Shows by performing the operation that subtraction is not commutative for interchange of the positive numbers, even though addition is.
 - b. Rewrites a given problem to illustrate the commutative or associative property of addition.
 2. Behaviors to be taught
 - a. Identifies which operations (whole numbers, numbers have the closure property)
 - b. Does problems showing that subtraction is not associative.
- D. Subtraction is a binary operation
 1. Prerequisites
 - a. Fills in frames for missing addend problems with two addends.
 - b. Supplies or completes the inverse problem in subtraction for a given addition problem.
 2. Behaviors to be taught
 - a. No new ones.
- E. Division with ten or greater
 1. Prerequisites
 - a. Review all elementary division facts.
 - b. Divide a 2 or more digit product by a 2 or 3 digit factor using the Algorithm. Checks on division problems by multiplication.
 - c. Prefer addition, subtraction, multiplication or division - using numbers to 1000, 10,000, 100,000.
 2. Behaviors to be taught
 - a. Illustrate that divisor is distributive over addition and subtraction when given a dividend or divisor.
 - b. Divide a two or more digit factor by using the Algorithm.
 - c. Solves more problems involving addition, subtraction, multiplication or division using numbers to 1,000, 10,000, 1,000,000.
- F. Average
 1. Prerequisites
 - a. Solves one step word problems involving combination of any arithmetic operation.
 - b. Solves division word problems to 200.
 - c. Estimates answers in addition for sum of 200 by rounding numbers to get a simple form.

2. Behaviors to be taught
 - a. Finds average of 2 or more numbers in numerical problem form.
 - b. Develops the rule for finding an average using simple problem situations.
 - c. Solves word problems requiring the finding of simple averages for quotients to 10.
3. Enrichment
 - a. Find radius and diameter.

G. Functions and relations

1. Prerequisites
 - a. Functions and relations are developed simultaneously with and/as an advanced activity in basic operations, sense of cardinality, number of theory, and mathematical structure.
 - b. Its prerequisites include any mathematical operations or relationships being investigated.
2. Behaviors to be taught
 - a. Finds the missing factor or product for multiplication problems with placeholders including letters. Uses or draws an array to find the missing factor or product. Products to 200.

H. Divisor less than 10, regrouping remainder

1. Prerequisites
 - a. Solves division problems by repeatedly subtracting multiple of divisor.
 - b. Uses the distributive principle to simplify division problems for 2 or 3 digit products.
2. Behaviors to be taught
 - a. Divisors and quotients to be taught
 - b. Divides a three or more digit product by a one digit factor.
 - c. Checks division by multiplication.
 - d. Solve equations with literal values which necessitate finding the greatest quotient and appropriate remainder.
3. Enrichment (circles)

I. Division - non closure - non commutative, non associative

1. Prerequisites
 - a. Uses the commutative principle for multiplication to solve problems with a 2 place factor.
 - b. Uses the associative principle in multiplying a 3 or more digit factor by 1,2,3 factors.
 - c. Identifies or selects from a list those number statements which illustrate a certain principle.
2. Behaviors to be taught
 - a. Illustrates that the commutative principle does not hold for division using small whole numbers.
3. Enrichment (none)

III. Optional Items

- A. Roman numerals
- B. Amount of Money
- C. Measurement, temperature, liquids, weight
- D. Finding perimeter
- E. Multiplication - multiplies 2 digits
- F. Graphs, line graph
- G. Geometry
- H. Fractional numbers

Grade Five
Primes Math

I. Items to be mastered

A. Multi-digit multiplication — two factors — regrouping

1. Prerequisite behavior
 - a. Uses multiplication algorithm for problems up to 10×10 .
 - b. Multiplies a one digit factor times a three or more digit factor.
 - c. Uses distributive principle to simplify multiplication of one and two digit factors.
2. Behaviors to be taught
 - a. Multiply using lattice method.
 - b. Multiplication by renaming — to 200.
 - c. Multiply using algorithm for multiples of powers of 10. e.g. 8×10 -80.
Also 3 digit numbers times 3 or more digit numbers.

B. Two combined operations

1. Prerequisites
 - a. Adds two or more addends
 - b. Performs subtraction problems to five digits.
 - c. Solves word problems using addition, subtraction, multiplication or division.
 - d. Divides — check by multiplication
2. Behaviors to be taught
 - a. Solves multi-step word problems.
 - b. Finds truth set for open sentences.
 - c. Makes truth set.

C. Division — Divisor less than 10, dividend regrouped, remainder.

1. Prerequisites
 - a. Solves division problems for dividends to 100.
 - b. Writes division remainder as a fraction.
 - c. Uses distributive principle to simplify division problems.
 - d. Supplies difference between a number and a product expressed as two factors.
2. Behaviors to be taught
 - a. Solves division problems by subtracting multiples of divisor.
 - b. Solves word problems to dividends of 200.
 - c. Behavior analysis in progress.

D. Division inverse of multiplication

1. Prerequisites
 - a. Solves multiplication problems to 200 using number line.
 - b. Solves division problems by performing repeated subtractions.
2. Behaviors to be taught
 - a. Solves multiplication and division numbers to 200.
 - b. Division by zero is undefined.
 - c. Writes the fractional equivalent for a division problem.

E. More than two factors regrouping

1. Prerequisites
 - a. Understands multiplication of zero.
 - b. Multiplies a one digit factor times a three or more digit factor.
2. Behaviors to be taught
 - a. Multiplies using associative principle for multiplication. Products to
 - b. Simplifies multiplication using associative principle.
3. Enrichment
 - a. None

- F. Division: divisor less than ten, dividend not regrouped remainder.
1. Prerequisites
 - a. Solves division problems by performing repeated subtractions.
 - b. Writes remainder of division problem as a fraction.
 2. Behaviors to be taught
 - a. Behavioral analysis in progress.
 3. Enrichment
 - a. None
- G. Multiplication = multiples of ten as a factor
1. Prerequisites
 - a. Multiplies using zero
 - b. Can multiply a one digit factor times a three or more digit factor.
 2. Behaviors to be taught
 - a. Uses multiples of ten to generalize multiplication and division facts.
 - b. Uses associative and distributive principle to simplify multiplication.
 - c. Rounds numbers to numbers between thousand —million.
 3. Enrichment
 - a. None
- H. Commutativity, a property of addition for fractions
1. Prerequisites
 - a. Subtraction is not the commutative or associative property of addition.
 - b. Adds fractions to sums of $\frac{4}{4}$ using picture aids.
 2. Behaviors to be taught
 - a. Uses closure property in adding fractions.
 - b. Uses commutative and associative principles in adding fractions.
 - c. Adds numbers with one or two decimal places.
 3. Enrichment
 - a. None
- I. Common fraction notation — equal denominators.
1. Prerequisites
 - a. Adds or subtracts any two fractions with equal denominators.
 - b. Behavior analysis in progress.
 2. Behaviors to be taught
 - a. Adds fractions with equal denominators.
 - b. Adds or subtracts mixed or improper fractions.
 - c. Adds two numbers with whole number parts and one or two decimal places.
 - d. One step word problems involving any fractions to twelfths.
 3. Enrichment
 - a. None
- J. Division: divisor less than ten, dividend regrouped, no remainder
1. Prerequisites
 - a. Solves division problems for dividends to 100.
 - b. Division is distributive over addition and subtraction.
 2. Behaviors to be taught
 - a. Estimates answers in division by rounding division, divided, or both.
 - b. Uses distributive principle to simplify division problems for two or three digit products.
 - c. Solves division word problems.
 - d. Does short division.
 3. Enrichment
 - a. None

- K. Subtraction (fractions) - inverse of adding fractions
 - 1. Prerequisites
 - a. Shows subtraction is inverse of addition.
 - b. Adds or subtracts any two fractions with equal denominators.
 - c. Column addition of fractions — equal or unequal denominators.
 - 2. Behaviors to be taught
 - a. Adds, subtracts, or multiplies fractions with unequal denominators.
 - b. Solves one step word problems involving fractions to twelfths.
 - c. Adds or subtracts improper and mixed fractions.
 - 3. Enrichment
 - a. None

- L. Subtraction of fractions — property of zero.
 - 1. Prerequisites
 - a. Subtraction is not commutative.
 - b. Adds or subtracts fractions with equal denominators.
 - 2. Behaviors to be taught.
 - a. Behavior analysis in progress.
 - 3. Enrichment
 - a. None

- M. Expanded notation for fractions
 - 1. Prerequisites
 - a. Writes numerals in expanded notation.
 - b. Identifies place value — numbers to 10,000.
 - 2. Behaviors to be taught
 - a. Reads decimal numbers.
 - b. Charts decimal numbers
 - c. Reads decimal fractions.
 - 3. Enrichment
 - a. None

- N. Empty set
 - 1. Prerequisite behaviors
 - a. Writes proper numeral when shown a set in N notation.
 - b. Behavior analysis in progress.
 - 2. Behaviors to be taught
 - a. Identifies cardinal number.
 - b. Behavior analysis in progress.
 - 3. Enrichment
 - a. None

- O. Read and write to one hundred billion
 - 1. Prerequisites
 - a. Identifies the number of tens and ones in pictures.
 - b. Writes numeral in expanded notation form.
 - 2. Behaviors to be taught
 - a. Writes numeral in expanded notation for a.
 - b. Identifies place value.
 - c. Writes standard numeral for a number represented as words.
 - d. Reads numbers to trillions.
 - 3. Enrichment
 - a. None

P. Recognize two-dimensional figures.

1. Prerequisites
 - a. Can identify a plane.
 - b. States or demonstrates what a plane is using points.
2. Behaviors to be taught
 - a. Identifies intersecting lines.
 - b. Identifies a plane
 - c. Discriminates between lines in a plane and lines not in a plane.
3. Enrichment
 - a. None

Q. Recognize three-dimensional figures

1. Prerequisites
 - a. Recognizes names of geometric figures.
 - b. Can identify geometric figures.
2. Behaviors to be taught
 - a. Knows and can select geometric figures or objects from a group of objects.
 - b. Finds surface areas of objects.
3. Enrichment
 - a. None

II. Items to be introduced

A. Addition with common fractions notation, unequal denominators.

1. Prerequisites
 - a. Adds or subtracts fractions with equal denominators.
 - b. Renames fractions in lowest terms.
2. Behaviors to be taught
 - Adds, subtracts, or multiplies fractions with unequal denominators.
 - a. Solves one-step word problems-fractions to twelfths.
 - b. Finds least common denominator.
3. Enrichment
 - a. None

B. Subtraction-fractions-unequal denominators

1. Prerequisites
 - a. Pittsburgh University Center - has content item.

C. Commutativity - multiplication - fractions

1. Prerequisites
 - a. Uses the commutative property of multiplication.
 - b. Uses the commutative and associative principles in adding fractions.
2. Behaviors to be taught
 - a. Multiplies fractions with factors of up to $\frac{4}{4}$ by whole numbers to 20.
 - b. Performs addition, subtraction, or multiplication with fractions with unequal denominators.
3. Enrichment
 - a. None

D. Multiplication of fractions developed from the number line.

1. Prerequisites
 - a. Adds or subtracts fractions equal denominators.
 - b. Adds two numbers with whole numbers parts and one or two decimal places.
2. Behaviors to be taught
 - a. Multiplies fractions with factors of up to $\frac{4}{4}$ by whole number factors up to 20.
 - b. Uses picture regions, number, lines, etc. when doing simple operations with unequal denominators.
3. Enrichment
 - a. None

- E. Multiplication-fractions-identity element
 - 1. Prerequisites
 - a. Uses \times sign in multiplication.
 - b. Shows 1 is identity element for multiplication.
 - 2. Behaviors to be taught
 - a. Understands dividing or multiplying fraction numerator or denominator by same number does not change fraction.
 - b. Can provide a fraction for a number pair.
 - c. Identifies other names for numbers.
 - 3. Enrichment
 - a. None

- F. Infinite sets
 - 1. Prerequisites
 - a. Identifies a set of whole numbers as a set beginning with zero.
 - b. Writes proper numeral when showing a set in N notation.
 - 2. Behaviors to be taught
 - a. List multiples of any number.
 - b. Uses braces to enclose members of a set.
 - 3. Enrichment
 - a. None

- G. Meaning of set and member
 - 1. Prerequisites
 - a. Can identify meaning of word set.
 - 2. Behaviors to be taught
 - a. Identifies subset
 - b. Behavior analysis in progress.
 - 3. Enrichment
 - a. None

- H. Finite set
 - 1. Prerequisites
 - a. Identifies cardinal number of a group.
 - b. Writes proper numeral when shown set in N notation.
 - 2. Behaviors to be taught
 - a. Writes proper numeral when shown a set in N notation.
 - 3. Enrichment
 - a. None

III. Optional Items

- A. Powers and exponents
- B. Intersection (Venn diagrams)
- C. Volume
- D. Averages
- E. Union of sets
- F. Measuring (linear)
- G. Liquids
- H. Rounding numbers
- I. Number sequences
- J. Percent
- K. Ratio
- L. Finding perimeter
- M. Money (crossword puzzle)
- N. Area II

Grade Six
Primes Math

I. Items to be mastered

A. Division of fractions the inverse of multiplying

1. Prerequisite behaviors
 - a. One is the identity element
 - b. Uses the multiplication algorithm
2. Behaviors to be taught
 - a. Identifies multiplying or dividing the numerator and denominator by the same number as being equivalent to multiplying or dividing the fraction by one.
 - b. Uses the division algorithm

B. Multiplication of fractions developed from addition of equal fractions.

1. Prerequisite behaviors
 - a. Solves multiplication problems for products to 200
 - b. Adds or subtracts any two fractions with equal denominators.
2. Behaviors to be taught
 - a. Multiplies a proper fraction by a whole number

C. Commutativity - multiplication - fractions

1. Prerequisite behaviors
 - a. Completes multiplication equation which illustrate commutative property.
 - b. Uses the commutative and associative principles in addition of fractions.
 - c. Identifies the relationship " $1/A$ or N "
2. Behaviors to be taught
 - a. Solves problems with zero in a factor
 - b. Applies the associative and commutative principle in multiplication.
 - c. Uses the multiplication algorithm
 - d. Behavioral analysis in progress

D. Addition - fractions - binary - operation

1. Prerequisite behavior
 - a. Adds or subtracts any two fractions with equal denominators.
 - b. Uses the commutative and associative principles.
 - c. selects those number statements which illustrate a principle
2. Behaviors to be taught
 - a. There are no behaviors for this item

E. Addition - fractions - unequal denominators

1. Prerequisite behaviors
 - a. Adds or subtracts any two fractions with equal denominators.
 - b. Uses the algorithm for addition and subtraction of fractions.
 - c. Performs column addition with two or more simple fractions.
2. Behaviors to be taught
 - a. Performs complex addition and subtraction.
 - b. Finds the least common multiple
 - c. Performs simple addition, subtraction or multiplication with fractions unequal denominators.

F. Decimal fraction notation

1. Prerequisite behaviors

- a. Completes problems which illustrate the commutative or associative property
- b. Adds two or more addends

2. Behaviors to be taught

- a. Adds two numbers with whole number parts.
- b. Adds two or more numbers with whole number parts.

G. Division of fractions: property of one

1. Prerequisite behaviors

- a. Finds the greatest common factor.
- b. Uses the multiplication algorithm for multiplying all fractions.

2. Behaviors to be taught

- a. Identifies multiplying or dividing the numerator and denominator by the same number as being equivalent to multiplying or dividing the fraction by one.

H. Common fraction notation

1. Prerequisite behaviors

- a. Performs simple addition, subtraction or multiplication with unequal denominators.
- b. Multiplies a proper fraction by a whole number.
- c. Uses the multiplication algorithm for multiplying fractions.

2. Behaviors to be taught

- a. Applies the associative or commutative principle in multiplication.
- b. Uses the multiplication algorithm.
- c. Performs more complex multiplication of fractions.
- d. Multiplies a proper fraction by a whole number. Reduces answer to lowest terms.

I. Equivalent common fractions

1. Prerequisite behaviors

- a. Uses objects, pictured objects or no pictured objects, to identify different fractional statements.
- b. Identifies one as the identity element for multiplication.
- c. Multiplies a proper fraction by a whole number.

2. Behaviors to be taught

- a. Identifies other names for numbers.
- b. Finds a set of equivalent fractions for a given fraction.
- c. Selects or writes fractions which depict a certain ratio or proportion.
- d. Performs simple ratio or proportion problems.

J. Ordering-fractions

1. Prerequisite behaviors

- a. Supplies the correct sign
- b. Uses the number line
- c. Performs simple ratio and proportion problems.

2. Behaviors to be taught

- a. Reads decimal fractions to millionths
- b. Supplies the correct sign $>$ or $=$
- c. Adds or subtracts fractions and whole numbers.
- d. Finds a set of equivalent fractions for a given fraction.

K. Subtraction of fractions developed in relation to subsets.

1. Prerequisite behaviors
 - a. Uses set braces to enclose members.
 - b. Identifies a subset of a given set by separating the set.
 - c. Identifies a subset by separating a specified number of sets by partitioning
2. Behaviors to be taught
 - a. Not in the computer printout.

II. Introductory items

A. Division - fractions - binary operation

1. Prerequisite behaviors
 - a. Uses the division algorithm
 - b. Identifies which number systems have closure property.
 - c. Identifies the associative, commutative, distributive principles.
2. Behaviors to be taught
 - a. Not in computer printout.

B. Noncommutativity, nonassociativity of division (fractions)

1. Prerequisite behaviors
 - a. Applies commutative principle in multiplication of fractions.
 - b. Applies associative principle in multiplication of fractions.
2. Behaviors to be taught
 - a. Performs addition, subtraction, multiplication or division for problems involving any common fractions.
 - b. Identifies the associative, commutative and distributive statements.
3. Enrichment
 - a. Worksheets on the principles.

C. Division of fractions closure

1. Prerequisite behavior
 - a. Uses the division algorithm for dividing proper, improper, complex fractions
 - b. Identifies which operations have the closure.
2. Behaviors to be taught
 - a. Not in computer printout.
3. Enrichment
 - a. Worksheets on the closure property.

D. Distributivity - fractions

1. Prerequisite behaviors
 - a. Shows the distributive property of multiplication over addition and subtraction.
 - b. Uses the multiplication algorithm.
 - c. Multiplies a decimal number times a decimal number.
2. Behaviors to be taught
 - a. Applies the distributive principle.
 - b. Uses the multiplication algorithm for multiplying all fractions.
3. Enrichment
 - a. None

E. Product sets to form new sets.

1. Prerequisite behaviors
 - a. Behavioral analysis in progress.
2. Behaviors to be taught
 - a. Behavioral analysis in progress.

- F. Partitioning sets to form new sets
 - 1. Prerequisite behaviors
 - a. Uses set braces to enclose members.
 - b. Selects a set which is a proper subset of a given set.
 - 2. Behaviors to be taught
 - a. Identifies a subset of a given set by separating the set.
- G. Zero not a divisor fractions
 - 1. Prerequisite Behaviors
 - a. Completes the inverse problem for division.
 - b. Completes the inverse problem for multiplication.
 - c. Uses the division algorithm.
 - 2. Behaviors to be taught
 - a. Not in computer printout.
- H. Equivalent percent notation
 - 1. Prerequisite behaviors
 - a. Finds a set of equivalent fractions for a given fraction.
 - b. Identifies other names for numbers.
 - c. Writes the decimal equivalent for simple fractions.
 - d. Changes decimal equivalent to fractions.
 - 2. Behaviors to be taught
 - a. Selects or writes fractions which depict a certain ratio or proportion.
 - b. Does problems requiring conversion from common fractions to percent.
- I. Numbers expressed in exponential form
 - 1. Prerequisite behaviors
 - a. Multiplies numbers in exponential form with same bases and positive powers.
 - 2. Behaviors to be taught
 - a. Not in computer printout
 - 3. Enrichment
 - a. Teacher constructed materials.
- J. Graphing - coordinate - plane
 - 1. Prerequisite behavior
 - a. Graphs the answer set of a statement.
 - b. Constructs number line and identifies the answer.
 - 2. Behaviors to be taught
 - a. Behavioral analysis in progress.
- K. Division of fractions: closure
 - 1. Prerequisite behavior
 - a. Uses the division algorithm.
 - b. Identifies a certain principle.
 - c. Identifies operations which have closure.
 - 2. Behaviors to be taught
 - a. Not in computer printout.
- L. Ordered pairs on a coordinate plane
 - 1. Prerequisite behavior
 - a. Graphs the answer set of a statement with literal variables.
 - b. Constructs a number line and identifies the answer on a number line.

2. Behaviors to be taught
 - a. Graphs lines and figures on a coordinate graph.
 - b. Gives the number pairs in matching two sets.
 - c. Makes a table of ordered pairs.
 - d. Plots ordered pairs of numbers on graph paper.
- M. Construct or draw three-dimensional figures
 1. Prerequisite behaviors.
 - a. Measures angles using a protractor.
 - b. Draws angles with a protractor.
 - c. Discriminates between lines in a plane and lines not in a plane.
 2. Behaviors to be taught
 - a. Identifies a triangular pyramid.
 - b. Constructs a triangular pyramid.
 - c. Identifies the surface area of prisms, cylinders, pyramids, cones.

III. Optional items

- A. Rounding for estimates
 1. The students will use the following materials
 - a. Paper
 - b. Ditto
 - c. Silver-Burdett text
- B. Addition of base five numerals
 1. Materials needed
 - a. Tape recorder
 - b. Holt text
 - c. Ditto
- C. Place value decimals
 1. Materials
 - a. Ditto
 - b. Place value board
- D. Angles (construction)
 1. Materials
 - a. Ditto
 - b. Tape recorder
 - c. Paper
 - d. Silver-Burdett text
- E. Associativity - directions are on the ditto
 1. Materials
 - a. Ditto
 - b. Paper
 - c. Pencil
 - d. Tape recorder
- F. Development of standard units - (Group work)
 1. Materials
 - a. Ditto
 - b. Teaching tape
 - c. Silver-Burdett text

- G. Temperature
 - 1. Materials
 - a. Text - Silver-Burdett
 - b. Teaching tape
 - c. Scale-therm
- H. English and metric units
 - 1. Materials
 - a. Silver Burdett text
 - b. Teaching tape
 - c. Ditto
- I. Basic concepts of percent
 - 1. Materials
 - a. Teaching tape
 - b. Silver-Burdett text
- J. Describe and analyze geometric figures
 - 1. Materials
 - a. Teaching tape
 - b. Silver-Burdett text
- K. Common fractions, unequal denominators
 - 1. Materials
 - a. Teaching tape
 - b. Silver-Burdett text
- L. Circles
 - 1. Materials
 - a. Teaching tape
 - b. Silver-Burdett text
- M. Probability
 - 1. Materials
 - a. Teaching tape
 - b. Houghton - text
- N. Clock arithmetic
 - 1. Materials
 - a. Teaching tape
 - b. Houghton - text
 - c. Clock face
- O. Finding perimeter of circumference
 - 1. Materials
 - a. Teaching tape
 - b. Silver-Burdett text
- P. Planes
 - 1. Materials
 - a. Teaching tape
 - b. Silver-Burdett text

- Q. Ordering of fractions
 - 1. Materials
 - a. Drill tape
 - b. Text
- R. Commutativity and associativity
 - 1. Materials
 - a. Drill tape
 - b. Text-Silver Burdett
- S. Historical systems of numeration
 - 1. Materials
 - a. Teaching tape
 - b. Houghton and Holt text